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On the Probability Distribution of Sea Level Changes in the Caspian Sea

Alexander Kislov[1]

Abstract—The Caspian Sea (CS) undergoes significant multi-scale variations in sea level. Based on empirical evidence (red noise-like behavior) and general ideas about temporal dynamical laws related to massive inertial objects, the observed changes of CS sea level represent a form of non-linear “self-induced” behavior. From this perspective, the mathematical model for this behavior is represented by the Fokker–Planck equation, the solution for which allows calculation of a probability distribution function (PDF) for CS sea level variations. For verification, the PDF is compared with an empirical histogram calculated using palaeohydrological data covering the last millennium. Despite the scatter, there are similarities between the two functions. In particular, both functions have a non-Gaussian asymmetric structure.

Keywords: Caspian sea, PDF of a sea level changes, Fokker–Planck equation.

1. Introduction

Climate variability exists at all timescales. Atmospheric variability, from seconds to decades, is described, synthesised, and summarised in the special Geophysical Research Letters collection (see Williams et al. 2017). Millennial-scale climate variability, known as the Milankovitch domain, is due to various direct and indirect astronomical influences (Berger and Loutre 1991). However, centennial-scale climate changes, manifested, for example, in the level/area changes of large lakes, ocean level variations, and large mountain glacier and ice sheet dynamics [see the special collection of the Past Global Changes magazine (Crucifix et al. 2017)], are poorly understood. The centennial scale is difficult to study as instrumental observations are often too short, timewise, and it is challenging to reconstruct data with the required temporal resolution. Rhythmic effects of solar forcing and volcanic activity on centennial scales are also little understood. Therefore, the internal dynamics of climate systems should be investigated to understand events on this scale. The dynamics of the Caspian Sea (hereafter referred to as CS) sea level and its area show variations on timescales of years, decades, centuries, millennia, and tens of millennia (Rychagov 1997; Yanina 2014; Krijgsman et al. 2019). The CS (36–47° N, 47–54° E) is the world’s largest lake (Fig. 1). Its mean sea level has varied between − 25 and − 29 m (relative to mean ocean level) over the last ∼ 100 years. Its main water source is the Volga River (80% of inflow), whose catchment area covers a large part of the Eastern European Plain. The water inflow is offset by evaporation over the CS. Its level is also affected by other rivers, precipitation over the sea, and subsurface runoff into the sea; however, their contributions are significantly lower, uncoordinated and irregular. The current CS sea level variability has been investigated extensively, (e.g., Rodionov 1994; Arpe et al. 2012; Bolgov et al. 2018). Instrumental data on CS sea level dynamics cover a period of slightly more than 100 years and, therefore, allow for an investigation of interannual and decadal sea level fluctuations. Paleo reconstructions are used to provide information about centennial sea level variations manifested by transgressive and regressive events. The majority of Holocene transgressions and regressions are part of the red noise family (Kislov 2018). From this, one can imply that the mathematical theory of Brownian motion can be used to reproduce the level variations. This means that there is a slow random response of the CS sea level position to the chaotically alternating accumulation of water budget anomalies. Another important feature is that the typical timescale of sea level dynamics is much larger

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Figure 1
Map of the Caspian Sea and its drainage basin outlined by a red line. The Volga River is main water source of the sea. The KBGB denotes the Kara Bogaz Gol Bay mentioned in the text

than the typical timescale of fluctuations in river runoff (Kislov 2016). A few pre-Holocene events do not fit the red-noise dependence. These are the so-called Khvalynian transgressions and the Last Glacial Maximum regression of the CS. These are major, unusual events; during these huge transgressive stages, for example, the Caspian Sea overflowed into the Black Sea. These events were caused by large background changes (ice sheets and permafrost conditions) causing precipitation river runoff, and evaporation rate changes. In the present papers, we apply the stochastic model to construct a probability distribution function (PDF) for CS sea level variability. The stochastic technique is not discussed in detail; we assume these general methods are sufficiently well known. The plan of the paper is as follows: second and third sections describe the approach and justification for a stochastic model. Section forth is devoted to the comparison of a theoretical PDF and an empirical PDF example. Finally, last section discusses our results and concludes the paper.

2. *Sea Level Fluctuations in the Caspian Sea: the Continuum of Scales*

Throughout geological history, the configuration of the seas and other characteristics have changed. Fluctuations in the level reflected in transgressive and regressive events are visible as was mentioned above at all scales. During huge transgressions the CS and the Black Sea sometimes connected, at that the flow of water always went from the CS to the Black Sea. However, this scaling should not be given much attention, since each scale has its own specific mechanisms for sea level variation. For example, it can be confidently assumed that with increasing scale, the contribution of geological processes increases to the variability. It should be stressed that the curves characterizing the geological history of the CS (Rychagov 1997; Krijgsman et al. 2019) are not plotted based on continuous series. In fact, it is a collection of events, interpolated among themselves. Moreover, transgressive events are recovered much more reliably than regressive ones. The most studied events are the Holocene and pre-Holocene postglacial age. Therefore, it makes sense to consider the issue of changing the level for this segment of paleo history. Changes in the level and area of the CS at this time occurred for hydroclimatic reasons almost without tectonic and other geological influences. During this period, there were several major transgressive and regressive events and many smaller events. Using the duration and scale of each anomaly, we estimated the spectrum and demonstrated that Holocene events belong to the red noise family (Kislov 2018). From this perspective, the CS sea level is represented by a system undergoing a random walk. It is multiscale dynamics can be described by the Ornstein–Uhlenbeck process that incorporates the action of precipitation and evaporation as a random white noise so that the whole can be thought of as an analogue of Brownian motion. Huge level changes during the pre-Holocene are not part of the red noise family. They could not have been provided by a mechanism of a random walk.

3. *Interpretation of the Caspian Sea Water Budget Equation as a Stochastic Equation*

The formalism used in the present study adapts the stochastic approach describing the random walk law to CS sea level dynamics. The first step is the formulation of an equation describing the water budget of any lake. Utilising the relation between the area (*f*) and level position (*h*) (hypsographic curve) in the linear form:

$$f = a + bh, \tag{1}$$

the water conservation equation can be transformed into an equation describing the change of lake level (Frolov 1985):

$$\frac{dh(t)}{dt} = -\left(\frac{\alpha}{a} - \frac{bV_{-,0}}{a^2}\right)h(t) - \frac{bV_{+,0}}{a^2}h(t) + \frac{V_+(t) - V_{-,0}}{a} - e(t). \tag{2}$$

Here, V_+ and V_- are, respectively, the contributions from river runoff and water outflow, and $V_{+,0}$ and $V_{-,0}$ are their currently estimated time-averaged values. The latter case is proposed to be the hydraulic formula: $V_-(t) = V_{-,0} + \alpha h(t)$. The evaporation minus precipitation over the sea area per unit of sea surface is given by $e(t)$. In shortened form, Eq. (2) can be defined as:

$$\frac{dh}{dt} = -\lambda h(t) + q. \tag{3}$$

The equation depicts that the river runoff and evaporation anomalies minus precipitation (concentrating in q) form a deviation from h. Negative feedback expressed by the two first terms of the right-hand side of Eq. (2) prohibit the development of large deviations. The variable, $(\lambda)^{-1}$, represents a spin-down time. This value is determined by parameters of the hypsographic curve of the CS and typical river runoff (mainly due to the Volga River) and water outflow from the CS to the Kara Bogaz Gol Bay (KBGB) (a closed lagoon on the eastern shore of the CS). For current conditions, $(\lambda)^{-1} \sim 20$ years, while the characteristic time of the $q(t)$ τ_q is slightly more than 1 year (see below). Because τ_q is much smaller that the characteristic time of the changes of h, we can consider Eq. (2) as a stochastic one, proposing that slow changes in h are balanced by the statistically averaged result of the fast fluctuations generated by q. This means that consecutive chaotic anomalies do not completely cancel each other out, and their residual effects accumulate. This process slowly forms a deviation that is controlled by spin-down processes.This interpretation, together with the red-noise behaviour of the CS sea level changes, allow us to represent Eqs. (2) or (3). as a stochastic Langevin equation (the foundation of the Brownian concept). From this perspective, the "force" $q(t)$ characterises as the delta-correlated stochastic process, and its autocorrelation function is approximated by:

$$K_q(\tau) = 2D_q\delta(\tau), \tag{4}$$

where τ is the time shift, $\delta(\tau)$ is the delta-function, and $D_q = \sigma_q^2\tau_q$. To avoid the influence of the low correlation of forcing (Frolov 1985) it was assumed that τ_q = 2 years. The value of σ_q^2 = 0.033 (m/year)^2 was calculated using river runoff and rate of evaporation data (Kislov and Toropov 2006).

The hypsographic curve of the CS consists of three intervals (Fig. 2) where parameters of Eq. (1) are different for each interval. The linear

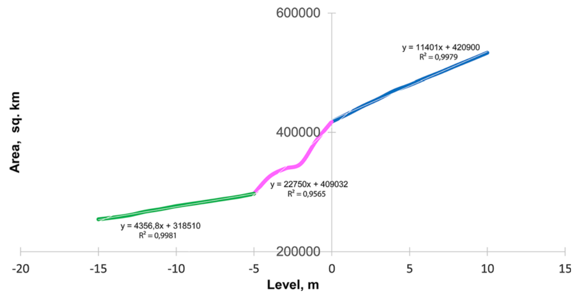

Figure 2
Hypsographic curve of the Caspian Sea area (km^2) vs. level position (m) (Kislov et al. 2014). White thin lines are linear approximations (see Eq.1) for each interval. R^2denotes the coefficient of determination

approximation in each interval is characterised by very high confidence (coefficients of determination are > 0.95). Subsequently, we assume $\lambda = \lambda(h)$ in Eq. (3).The function $\lambda(h)$ can be expressed analytically in the following form:

$$\lambda(h) = \lambda_1 + (\lambda_2 - \lambda_1)e_{h>-\xi} + (\lambda_3 - \lambda_2)e_{h>0}, \quad (5)$$

where (using rounded values) λ_1= 0.01 year^{-1}, λ_2= 0.05 year^{-1}, and λ_3= 0.02 year^{-1}; zero and ξ = 5 m are two inflection points on the CS hypsographic curve, where its parameters denote sharp changes; e depicts the unit step function.

We assume that changes in λ are determined in each interval by a and b, as well as $V_-(t)$. Changes in $V_+(t)$ are not explicitly taken into account here, despite these affecting CS sea level changes. A direct calculation shows that an increase or decrease in λ by a factor of 1.5, due to variations in runoff volume, cannot remove the differences between the hypsographic curve intervals. Dependence of λ on $V_-(t)$ reflects the role of the KBGB (see Fig. 1). The flow of CS water cascades down into the KBGB and is balanced by substantial evaporation. Currently, water discharge to the KBGB controls the duration and amplitude of the CS sea level anomalies. When the CS sea level rises significantly, the KBGB transforms into an ordinary bay, with the spin-down mechanism of the KBGB ending. Conversely, when the CS sea level decreases, the KBGB dries out, forming land. This, again, stops the spin-down role of the KBGB. The problem denoted by Eqs. (3–5) can be formulated in the form of the Fokker–Planck equation describing the PDF:

$$\frac{\partial p(h,t)}{\partial t} - \frac{d}{dh}(\lambda(h)hp(h,t)) = D\frac{\partial^2 p(h,t)}{\partial h^2}. \quad (6)$$

Consider the steady-state variant of Eq. (6):

$$\frac{dp}{dh} = -\frac{\lambda(h)hp(h,t)}{D}; \quad (7)$$

its solution is the following:

$$p = p_0 exp\left(-\frac{h^2}{2D/\lambda}\right), \begin{cases} \lambda = \lambda_1, h < -\xi; \\ \lambda = \lambda_2, -\xi \le h < 0; \\ \lambda = \lambda_3, h > 0 \end{cases} \quad (8)$$

The constant p_0 is determined from the definition of total probability

$$\int_{-\infty}^{\infty} pdh = 1. \quad (9)$$

Hence, we calculate

$$\frac{1}{P_0} = \int_{-\infty}^{-\xi} exp\left(-\frac{h^2}{2D/\lambda_1}\right)dh + \int_{-\xi}^{0} exp\left(-\frac{h^2}{2D/\lambda_2}\right)dh + \int_{0}^{\infty} exp\left(-\frac{h^2}{2D/\lambda_3}\right)dh. \quad (10)$$

In accordance with the properties of the exponential function, there is no need to consider events located further than approximately three standard deviations ($3\sigma_h = 3\sqrt{D/\lambda}$) from the mean. For example, for a high CS sea level ($h \gg 0$), $\lambda = \lambda_3$ and $3\sigma_h = 5.5$ m. Therefore, if mean CS sea level position is far from zero (– 28.5 m), the time variations are the Gaussian random noise.

4. Comparison of PDF and Empirical Histogram

To check the expression (8), its curve should be compared with an empirical histogram. An empirical PDF of the CS sea level fluctuations can be calculated based on instrumental records. However, this assessment may be incorrect because dependent samples (highly correlated annually averaged data) were used for the statistical analysis. Additionally, a 2000-year series, consisting of a sequence of 20-years averaged quantities of reconstructed CS sea level data, was used to calculate the PDF. The PDF can be represented by a Gaussian curve (Kislov 2016). However, due to new data, the reliability of this reconstruction is in doubt. In this paper, an empirical histogram was calculated based on a 1050-year series, including a sequence of 25-years averaged quantities of reconstructed CS sea level data (Kislov et al. 2014). During this period, the CS sea level moved from the Derbentian regression stage via the fifth phase New-Caspian transgression stage to the current stage. Such a series is not entirely convenient for statistical analysis as it includes long-period variation. However, no other series has the required data discreteness and sufficient length. The mean CS sea level during this period was calculated as − 27.8 m; the standard deviation was 2.2 m. Combining Eqs. (8) and (10) we have, for theoretical calculation of the PDF, the next expression:

$$p(h) = \frac{2}{\sqrt{2\pi D/\lambda_2} + \sqrt{2\pi D/\lambda_3}} \begin{cases} exp\left(-\frac{h^2}{2D/\lambda_2}\right), h \leq 0 \\ exp\left(-\frac{h^2}{2D/\lambda_3}\right), h > 0 \end{cases} \quad (11)$$

This formula contains information from only two intervals of the CS hypsographic curve because the mean position of the CS sea level (− 27.8 m) is located near the right inflection point. Information from the left side is not included because the CS sea level lies far from the left inflection point (− 5 m). This curve (Eq. 11) is a combination of two different Gaussian curves and asymmetrically extends towards a higher CS sea level (Fig. 3). Despite the scatter, the PDF and histogram have the same features. In particular, the asymmetry of the histograms toward higher CS sea levels.

5. *Discussion and Conclusion*

The investigation of centennial changes in the CS sea level is important for properly assessing the contribution of anthropogenic sources and future climate projections. Additionally, the study of CS sea level changes is important for a greater understanding of regional-scale climate variability as the CS water is sourced from the large East European Plain. Instrumental data from the CS cover a period of slightly more than 150 years and, therefore, do not allow the study of some secular variations. The study of centennial CS sea level variations during the Holocene (recent ∼ 11,000 years) are ideal due to no abrupt climate changes during this period. However, continuous series depicting level changes are absent, data sets are practically a collection of events, interpolated among themselves.In terms of the theoretical model, it was proposed that the CS sea level dynamics can be described as a random walk. Due to the multiscale character of fluctuations and their random origin, the theory of random processes is useful. From this perspective, the CS sea level change model is described by the Langevin equation that incorporates the action of precipitation, evaporation, and river runoff as a random white noise. The transition to the Fokker–Planck equation allows the calculation of a PDF for the CS sea level variations. For verification, the PDF was compared with an empirical histogram. Despite the scatter, there are similarities between the two functions. The non-Gaussian character of the PDF could be a sign of specific behaviour in system dynamics (Sardeshmukh and Sura 2009; Franzke et al. 2012) and we consider this behaviour is determined by the feedback that exists between the level dynamics and the changing of damping property of the system connected to the role of the KBGB. It can be assumed that this mechanism was characteristic of the entire Holocene. Now let's turn to the huge CS sea level changes. In the era under study the level was in the maximum of the Early Khvalynian transgression (+ 75 m relative to the modern stage, age = 25–35 ka BP, ∼ MIS3), in the minimum of the Enotaevka regression (− 50 m relative to the modern stage, age = 18–25 ka BP, the Last Glacial Maximum—LGM), in the maximum of the Late Khvalynian transgression (+ 27 m relative to the modern stage, age = 14–18 ka BP) (Yanina 2014; Krijgsman et al. 2019). They were accompanied by substantial changes in the area of the CS in its northern part, where depth is shallow. Such huge changes in the level, from the point of view of the PDF, have a very small probability (Fig. 3). Using an average return period, we can calculate that such anomalies take tens of thousands of years to appear. The climate system has not been stable for so long, so such a recurrence period characterizes an incredible event. On the other hand, the fact that large anomalies

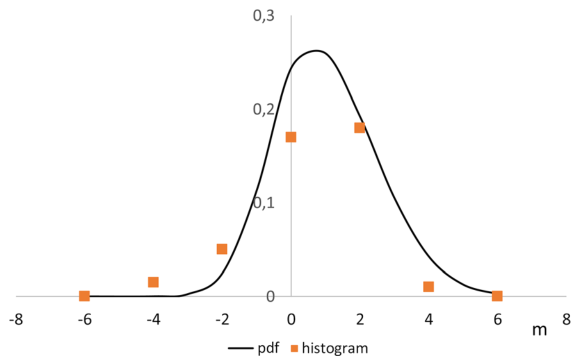

Figure 3
Comparison of the probability distribution function of the Caspian Sea level changes (see expression 11) and empirical histogram

have occurred in the history of the CS suggests that the random walk mechanism is not suitable for them. In such circumstances, it is possible to study the mechanism of individual events. A deep minimum of the Enotaevka regression is explained by a large decrease in precipitation over the East European Plain, which was regionally influenced by the Scandinavian ice sheet and by a generally influenced by decrease in evaporation from oceans during the LGM. The models of the Paleoclimate Modelling Intercomparison Project (PMIP) gave correct estimates of a decrease in the Volga runoff and consequently a decrease in the CS sea level (Kislov and Toropov 2006, 2011). The next examples are the Khvalynian transgressions (Early and Late stages) accompanied by the CS overflowing into the Black Sea. These are clear evidence based on reconstructions (Chepalyga 2007; Yanina 2014; Krijgsman et al. 2019). However, it is still unclear, what was the source of the "additional" water needed to support such an ultra-high CS sea level. Attempts to solve the "additional" water paradox have so far been unsuccessful (Yanko-Hombach and Kislov 2017). Based on the materials of the article, we can conclude that, at least, the origin of these anomalies as a result of random walks is completely excluded.

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