Meander of a Barotropic Zonal Current Crossing a Bottom Ridge (Periodic Regime)

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The interaction of a fluctuating barotropic zonal flow with an infinite bottom ridge is examined in the quasi-geostrophic approximation on the β plane. The solution of the equation for pressure perturbation is constructed by direct asymptotic expansion with respect to the small parameter, proportional to the amplitude of fluctuations of the principal flow. The first approximation of Green's function for easterly and westerly flows is analyzed. The feasibility of employing this model for explaining features in the time variability of the antarctic circumpolar current is discussed.

The question of the effect of bottom-topography irregularities in the form of an infinite depression, ridge or trough on the steady-state barotropic zonal flow was examined by Clarke and Fofonoff [9], McIntyre [11], Porter and Rattray [12] and Vaziri and Boyer [14]. Much less work was done for investigating unsteady modes (transients as well as periodic); mention should be made here of work by Robinson and Gadgil [13] and by the first of the present authors [4].

In this paper we consider the problem of meander of an oscillating zonal current above a non-moving arbitrarily oriented infinite ridge; the solution is periodic in time. The use of perturbation with respect to the small amplitude of fluctuations of the principal flow makes it possible to obtain corrections to the Green function for flows with easterly and westerly directions in explicit form.

Statement of the problem. On the assumption that the relative perturbation of the bottom topography is of the order of the Rossby number, the equation of potential vorticity in the quasi-geostrophic approximation for the barotropic ocean on the β plane in nondimensional form can be written as [4]

$$\gamma\Delta p_t + J(p, \Delta p + y + \sigma h) + k\Delta p = 0. \quad (1)$$

Here Δ and J are horizontal Laplace and Jacobi operators; the zonal and meridional coordinates x and y are scaled using $L^* = \sqrt{U^*/\beta}$ where U^* is the characteristic horizontal velocity, whereas β is the Rossby parameter; $\gamma = 1/(T^*\sqrt{\beta U^*})$, where T^* is the characteristic time scale t; topographic parameter $\sigma = h^* L^* \Omega^*/H^* U^* = O(1)$, where h^* is the amplitude of the projection of bottom topography, H^* is the mean ocean depth and Ω^* is the characteristic value of the Coriolis parameter; friction parameter $k = (L^*/H^* U^*)\sqrt{\nu\Omega^*/2}$, where ν is the kinematic coefficient of vertical eddy viscosity in the bottom boundary layer; the pressure perturbation scale is taken equal to $\rho^*\Omega^* U^* L^*$, where ρ^* is the constant density.

In the absence of bottom irregularities (σ=0) Eq. (1) permits the solution $p = -U(t)y$, corresponding to zonal flow with velocity $U(t)$, which is then assumed to be of constant sign. We assume a "cylindrical" perturbation of the bottom topography in the form $h = h(\xi)$, $\xi = x\cos\alpha - y\sin\alpha$, where, unlike in [4], ξ has the meaning of the distance along the normal to the isobath ξ=0, inclined at angle α to the meridian, $|\alpha| < \pi/2$. Setting

$$p = -U(t)y + \sigma\psi(\xi, t), \quad (2)$$

for the perturbation pressure field we obtain from (1) the expression

$$2\mu\psi_{\xi\xi t} + \psi_\xi + U(t)\psi_{\xi\xi\xi} + 2\varkappa\psi_{\xi\xi} = -U(t)h'(\xi), \quad (3)$$

where

$$\mu = \gamma/2\cos\alpha, \; \varkappa = k/2\cos\alpha. \quad (4)$$

Note that the identical vanishing of nonlinear terms in Eq. (3) at arbitrary σ is the result of the fact that ψ and h are a function of only a single space coordinate (ξ).

Since in the presence of friction and finite $h(\xi)$ the pressure perturbations must decay at infinity, single integration of Eq. (3) yields

$$2\mu\psi_{\xi t} + \psi + U(t)\psi_{\xi\xi} + 2\varkappa\psi_\xi = -U(t)h(\xi). \quad (5)$$

The Cauchy problem for Eq. (5) was analyzed in detail by the first of the present authors [4], the mixed problem was considered by Robinson and Gadgil [13]. Below we shall consider only periodic solutions of this equation at specified periodic function $U(t)$, $U(t+2\pi) = U(t)$ which means that the period in dimensional variables is equal to $2\pi T^*$.

Results are interpreted best in terms of streamlines. For the streamline which asymptotically approaches the x axis upstream, we have from (2) that $-U(t)y + \sigma\psi(x\cos\alpha - y\sin\alpha, t) = 0$. In the particular case of meridional isobaths (α = 0)

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we obtain in explicit form

$$y=\sigma\psi(x, t)/U(t). \qquad (6)$$

The solution of Eq. (5) is most conveniently expressed in terms of Green's function:

$$\psi(\xi, t)=\int_{-\infty}^{\infty} G(\xi-\xi', t) h(\xi') d\xi',$$

which satisfies the equation

$$2\mu G_{\xi t}+G+U(t) G_{\xi\xi}+2\varkappa G_{\xi}=-U(t) \delta(\xi) \qquad (7)$$

and conditions of decay at infinity; here $\delta(\xi)$ is the Dirac delta function.

Construction of the formal periodic solution. Representing Green's function in the form of the Fourier integral

$$G(\xi, t)=\frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{G}(\omega, t) e^{-i\omega\xi} d\omega, \qquad (8)$$

we obtain from Eq. (7) the following expression for the transform $\bar{G}$

$$2\mu\bar{G}_t+\left[2\varkappa+i\left(\frac{1}{\omega}-U(t)\omega\right)\right]\bar{G}=-iU(t)\omega. \qquad (9)$$

We first consider the case of long-period fluctuations of the velocity of undisturbed flow, when $\mu\ll 1$. From Eq. (9) we have approximately

$$\bar{G}=U(t)/U(t)\omega^2+2i\varkappa\omega-1,$$

whence it follows

$$G=-\frac{\theta(\xi) U(t)}{\sqrt{U(t)-\varkappa^2}} e^{-\varkappa\xi/U(t)} \sin(\xi\sqrt{U(t)-\varkappa^2}/U(t)),$$

$$U>\varkappa^2 \qquad (10)$$

$$G=-\frac{\theta(\xi) U(t)}{\sqrt{\varkappa^2-U(t)}} e^{-\varkappa\xi/U(t)} \operatorname{sh}(\xi\sqrt{\varkappa^2-U(t)}/U(t)),$$

$$0<U<\varkappa^2 \qquad (11)$$

$$G=\frac{U(t)}{2\sqrt{\varkappa^2-U(t)}} e^{-(\varkappa\xi-|\xi|\sqrt{\varkappa^2-U(t)})/U(t)}, \quad U<0, \qquad (12)$$

where $\theta(\xi)$ is a unit Heaviside function. The resultant solutions have the same form as in the case of the steady-state problem (U = const), but with parametric time dependence. In the absence of friction ($\varkappa=0$) and for a δ shaped meridional range ($\alpha=0$) the form of the perturbed flow is, according to Eq. (6), given by the expressions

$$y=-\frac{\sigma\theta(x)}{\sqrt{U(t)}} \sin(x/\sqrt{U(t)}), \quad U>0$$
$$y=\frac{\sigma}{2\sqrt{-U(t)}} \exp(-|x|/\sqrt{-U(t)}), \quad U<0, \qquad (13)$$

In the easterly flow ($U(t)>0$) a "leeward" Rossby wave forms behind the ridge; the length and amplitude of this wave change respectively in direct and inverse proportion to $\sqrt{U(t)}$; the flow upstream of the ridge is undisturbed. In westerly flow ($U(t)<0$) the perturbations are symmetric relative to the ridge, decaying exponentially with distance from it. The maximum shift of the streamlines at $x=0$ is inversely proportional to $\sqrt{-U(t)}$.

In the general case the periodic solution (with period 2π) of Eq. (9) is

$$\bar{G}=\frac{1}{2\mu\omega^3}\left\{\frac{2\varkappa\omega+i}{E(t)}\left[\frac{1}{1-E(2\pi)}\int_0^{2\pi} iE(\tau) d\tau-\int_0^t E(\tau) d\tau\right]+2\mu\omega\right\},$$

where

$$E(t)=\exp\left\{\frac{1}{2\mu}\left[2\varkappa t+i\left(\frac{t}{\omega}-\omega S(t)\right)\right]\right\}, \quad S(t)=\int_0^t U(\tau) d\tau.$$

The method of perturbations for oscillating flow. Let

$$U(t)=U_0+\varepsilon f(t), \qquad (14)$$

where $\int_0^{2\pi} f(t) dt=0$, $0<\varepsilon\ll I$ and $U_0=\pm 1$, respectively for the easterly and westerly flows. Then

$$S(t)=U_0 t+\varepsilon F(t), \quad F(t)=\int_0^t f(\tau) d\tau, \text{ here } F(0)=F(2\pi)=0.$$

Expanding the exponents in a power series of small parameter ε, we can easily obtain

$$G=\sum_{m=0}^{\infty}\left(\frac{\varepsilon}{2\mu}\right)^m \frac{\bar{G}_m}{m!} \qquad (15)$$

where

$$\bar{G}_0=U_0 U_0^2+2i\varkappa\omega-1,$$

$$\bar{G}_m=\frac{(-i\omega)^{m-3}(1-2i\varkappa\omega)}{2\mu E_0(t)}\left\{\frac{1}{E_0(2\pi)-1}\int_0^{2\pi} E_0(\tau)[F(\tau)-F(t)]^m d\tau+\right.$$
$$\left.+\int_0^t E_0(\tau)[F(\tau)-F(t)]^m d\tau\right\}, \quad m\geqslant 1,$$
$$E_0(t)=\exp\left\{\frac{t}{2\mu}\left[2\varkappa+i\left(\frac{1}{\omega}-\omega U_0\right)\right]\right\}.$$

The integrals are easily evaluated at $f(t)$ = const, which corresponds to $F(t)$ = sin t. Since

$$\int E_0(\tau)(\sin\tau-\sin t)^m d\tau=E_0(\tau)\varphi_m(\tau, t), \quad m\geqslant 1,$$

where always $\varphi_m(\tau+2\pi, t)=\varphi_m(\tau, t)$, it can be shown that

$$\bar{G}_m=\frac{(-i\omega)^{m-3}(1-2i\varkappa\omega)}{2\mu}\varphi_m(t, t), \quad m\geqslant 1.$$

In particular, for m = 1 we have

$$\bar{G}_1=\frac{2\mu(1-2i\varkappa\omega)[(1-\omega^2U_0-2i\varkappa\omega)\cos t-2\mu\omega i\sin t]}{(\omega^2U_0+2i\varkappa\omega-1)[(\omega^2U_0+2i\varkappa\omega-1)^2-4\mu^2\omega^2]}.$$

Inverting the Fourier transform, we obtain

$$G(\xi, t)=G_0(\xi)+\frac{\varepsilon}{2\mu}G_1(\xi, t)+O\left(\left(\frac{\varepsilon}{2\mu}\right)^2\right).$$

We write the results separately for the easterly and westerly flows.

$$\text{a) } U_0 = 1;\ G_0(\xi) = -\theta(\xi)e^{-\varkappa\xi}\frac{\sin R\xi}{R},$$

$$G_1(\xi,t) = \theta(\xi)\Big\{e^{-\varkappa\xi}\Big(\cos R\xi - \frac{\varkappa}{R}\sin R\xi\Big)\sin t + \frac{1}{2}e^{-(\varkappa-\varkappa_1)\xi}[A_1\cos(t-(\mu+\mu_1)\xi) + (B_1-1)\sin(t-(\mu+\mu_1)\xi)] - \frac{1}{2}e^{-(\varkappa+\varkappa_1)\xi}[A_1\cos(t+(\mu_1-\mu)\xi) + (B_1-1)\sin(t+(\mu_1-\mu)\xi)]\Big\}, \qquad (16)$$

where

$$R = \sqrt{1-\varkappa^2},\quad A_1 = \frac{\varkappa\mu_1 - \mu\varkappa_1}{\mu_1^2+\varkappa_1^2},\quad B_1 = \frac{\mu\mu_1 + \varkappa\varkappa_1}{\mu_1^2+\varkappa_1^2},$$

$$\binom{\mu_1}{\varkappa_1} = \Big\{\frac{1}{2}[\sqrt{(1+\mu^2-\varkappa^2)^2+4\mu^2\varkappa^2} \pm (1+\mu^2-\varkappa^2)]\Big\}^{1/2},$$

here conditions $\mu_1>\mu$ and $\varkappa\geqslant\varkappa_1$ are satisfied everywhere, and equality is attained at $\varkappa=\varkappa_1=0$.

$$\text{b) } U_0 = -1;\ G_0(\xi) = \frac{1}{2Q}e^{\varkappa\xi - Q|\xi|},$$

$$G_1(\xi,t) = -\frac{1}{2}\Big(\frac{\varkappa}{Q}\pm 1\Big)e^{(\varkappa\mp Q)\xi}\sin t - \frac{1}{2}e^{(\varkappa\mp\varkappa_2)\xi}\times[A_2\cos(t+(\mu\pm\mu_2)\xi) + (B_2\mp 1)\sin(t+(\mu\pm\mu_2)\xi)],\quad \xi \gtrless 0, \qquad (17)$$

where

$$Q = \sqrt{1+\varkappa^2},\quad A_2 = \frac{\mu\varkappa_2 + \varkappa\mu_2}{\mu_2^2+\varkappa_2^2},\quad B_2 = \frac{\mu\mu_2 - \varkappa\varkappa_2}{\mu_2^2+\varkappa_2^2},$$

$$\binom{\mu_2}{\varkappa_2} = \Big\{\frac{1}{2}[\sqrt{(1+\varkappa^2-\mu^2)^2+4\mu^2\varkappa^2} \pm (\mu^2-\varkappa^2-1)]\Big\}^{1/2},$$

here $\varkappa\leqslant\varkappa_2$ and $\mu\geqslant\mu_2$.

In the particular case of absence of friction, the above formulas become:

$$\text{a) } U_0=1;\ G_0(\xi) = -\theta(\xi)\sin\xi,$$

$$G_1(\xi,t) = \theta(\xi)\Big[\cos\xi\sin t - \cos(\mu_1^{(0)}\xi)\sin(t-\mu\xi) - \frac{\mu}{\mu_1^{(0)}}\sin(\mu_1^{(0)}\xi)\cos(t-\mu\xi)\Big],$$

where now $\mu_1^{(0)} = \sqrt{1+\mu^2} > \mu$.

$$\text{b) } U_0 = -1;\ G_0(\xi) = \frac{1}{2}e^{-|\xi|},$$

$$G_1(\xi,t) = \mp\frac{1}{2}\Big[e^{-|\xi|}\sin t - \Big(1\mp\frac{\mu}{\mu_2^{(0)}}\Big)\sin(t+(\mu\pm\mu_2^{(0)})\xi)\Big],\quad \xi\lessgtr 0$$

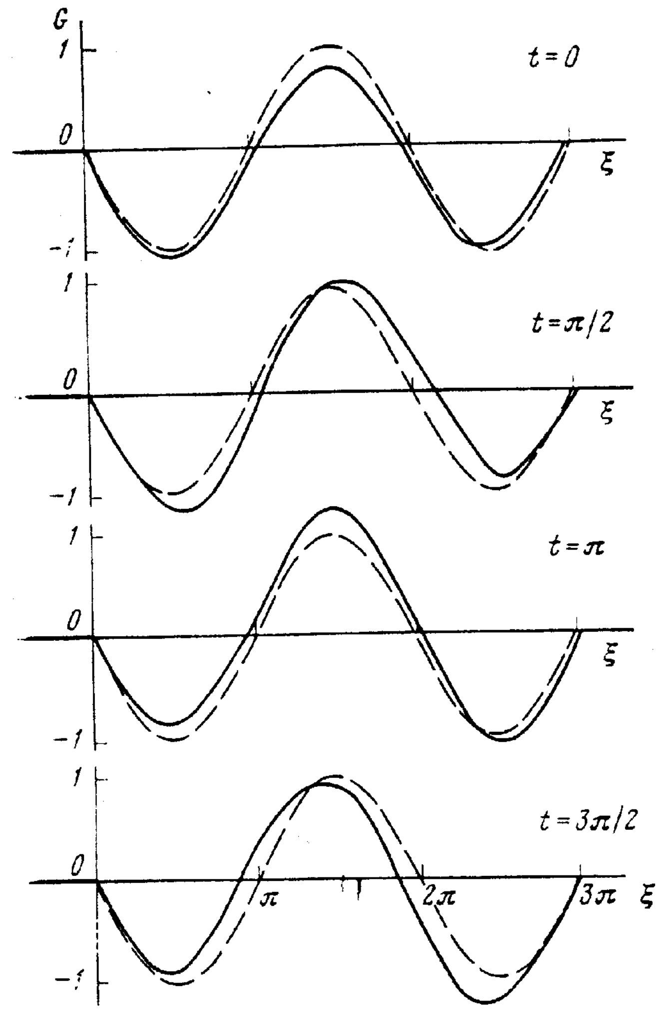
at $\mu>1$, $\mu_2^{(0)} = \sqrt{\mu^2-1} < \mu$ and

$$G_1(\xi,t) = \mp\frac{1}{2}\Big\{e^{-|\xi|}\sin t - e^{-\varkappa_2^{(0)}|\xi|}\Big[\sin(t+\mu\xi) \mp \frac{\mu}{\varkappa_2^{(0)}}\cos(t+\mu\xi)\Big]\Big\},\quad \xi\lessgtr 0$$

at $\mu<1$, $\varkappa_2^{(0)} = \sqrt{1-\mu^2}$.

Qualitative analysis of Green's function. Asymptotic expansion (15) is not uniformly suitable with respect to parameter μ, and is valid only at $\varepsilon\ll 2\mu$. The case of excessively small ε is of little interest and it hence should be assumed that at least $\mu\geqslant O(1)$ (the approximate solutions at $\mu\ll 1$ are defined by Eqs. (10)-(12)). This imposes restrictions on the possible periods $2\pi T^*$ of fluctuations of the main flow. Setting $U^* = 10$ cm/sec, $\beta = 2\cdot 10^{-13}$/cm-sec and $\alpha = 0$, we obtain from Eq. (4) the estimate $2\pi T^* \leqslant \pi/\sqrt{\beta U^*} = 2{,}22\cdot 10^6$ sec $\approx$ 25 days, which is close to the synoptic period.

It follows from Eq. (16) that no perturbations are ever present upstream of the obstacle in easterly flow, whereas downstream it is composed of a set of standing and modulated traveling waves.



Behavior of Green's function for the easterly flow without friction at different times. The solid curve corresponds to the approximate unsteady-state solution $G=G_0+\frac{\varepsilon}{2\mu}G_1$ in the case of $\mu=1,\ \varepsilon=0.5$; the dashed curve corresponds to the steady-state solution $G=G_0$.

The figure depicts the behavior of $G=G_0(x)+\frac{\varepsilon}{2\mu}G_1(x,t)$ at $\varkappa=0$, $\mu=1$, and $\varepsilon=1/2$ for easterly flow as compared with the steady-state solution $G_0(x)$ (dashed curve) each one quarter of the period at times $t=0, \pi/2, \pi$, and $3\pi/2$. Even if a rather high value of ε is assumed, the differences are relatively moderate. This means that at $\mu>\varepsilon/2$ the streamlines can be calculated from the approximate formula $-U(t)y+\sigma\psi_s(\xi)=0$, where $\psi_s(\xi)$ is the solution of the steady-state problem. At $\alpha = 0$ we obtain for the streamlines

$$y \approx \frac{\sigma}{U(t)}G_0(x) = -\frac{\sigma\theta(x)\sin x}{U(t)},$$

i.e., the amplitude of the virtually standing wave varies in inverse proportion to the velocity of unperturbed flow (compare with Eq. (13) for small μ).

It is interesting to note that in the second approximation, $G_2(\xi, t)$, the expression for which is not presented due to its cumbersomeness, there appears the steady-state term $\theta(\xi)\sin\xi$ as a result of interaction between different modes for even approximations in equation (7) with variable coefficients.

In the westerly flow modulated traveling waves appear both down and upstream. The amplitude of their modulation in the case of $\varkappa=0$ increases beyond bounds as $\mu\to1$ ($\mu_2^{(0)}\to0$, $\varkappa_2^{(0)}\to0$) and no bound periodic solutions exist at $\mu = 1$. An analogous resonant phenomenon occurs also for $G_2(\xi, t)$ at $\mu = 1/2$. Apparently, this points to instability of the westerly flows at $0<\mu<1$.

Possible applications. It appears to us that the above model can be useful for analyzing features in the time variability of leeward Rossby waves, forming in powerful zonal flows, crossing bottom ridges and troughs in their path. The antarctic circumpolar current (ACC) is a classical example of such a flow. Although there is still no concensus concerning the spatial structure of the ACC, according to studies by Ganson et al. [2] and Sarukhanyan [7], which, in our opinion are most representative, it is an easterly current, present in virtually the entire layer of water from the surface to the bottom and having (at least 200 m below the surface) a monotonically decreasing velocity profile. Unfortunately, the currently available information on the time variability of ACC are very scarce. Thus, Ivanov [3] and Savchenko et al. [6] suggest that the year-to-year variability of the flow rate in the ACC is insignificant, whereas Treshnikov et al. [8] estimate its multi-annual variability at 20 percent of the mean. Analysis of the charts of dynamic topography from data of approximately 1000 hydrological stations of various expeditions in the Drake Passage [10] yields an estimate of 10 to 15 percent for changes in the flow rate. According to Ivanov [3], who used the data of the Discovery II expedition in 1938-1939, seasonal variations in the flow rate in the ACC along 20°E comprise approximately 30 percent. Finally, Vladimirov and Savchenko [1] note statistically steady fluctuations of the time variation in current velocity components along 132°E at all depths with a period of 2 to 3 days; the short-period variability of surface circulation in the region of 160°E is pointed out by Moroz [5].

Judging by the above data, the velocities of the ACC undergo time variations at different scales—from several days to several years, however, they as a rule are small. This justifies the use of simplifications of the analytic model, based on (14).

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