HEAD-ON COLLISIONS OF DISTRIBUTED HETONS[1]

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1. We know [1] that if point geostrophic vortices located in a different layers of a two-layer liquid do not overlap in plan, have the same strengths but differ in sign, then they behave like vortex pairs: i.e., in the absence of external forces, they move uniformly along straight lines (case of translational steady state). The fundamental difference between such a two-layer set of single vortices and a vortex pair situated in the same layer is the monotonic dependence of the velocity V on the distance a between its vortices [1, 2]: while in the first case V tends to zero as $a \to 0$ and $a \to \infty$, in the second case, for small a, $V(a) \sim a^{-1}$. Similar properties are exhibited by distributed two-layer vortex structures in which the centers of the top and bottom vortices do not overlap in plan and which exhibit a zero strength in the potential-vorticity tubes [3]. Baroclinic vortices of this type belong to the group of the so-called hetons, which have recently been studied both theoretically in connection with single geostrophic vortices [1, 2, 4] and in laboratory experiments [5, 6]. Interaction of the vortices is the aspect that has been least studied. The hypothetical treatment of hetons [2] as a possible oceanologic phenomenon suggests that the interaction of two-layer vortices is of practical as well as theoretical interest.

In the present paper we describe numerical experiments, using the contour dynamics technique, on head-on collisions of distributed hetons, which, among other things, have revealed a new steady state consisting of a quasielliptical vortex, formed by the coalescence of two vortices in one layer, plus two peripheral vortices in the other layer that rotate with the quasielliptical vortex around a common center of gravity.

2. Consider the model of a two-layer ocean with a solid lid on the surface and a smooth, level bottom. Let ρ_1, H_1, ρ_2 and H_2 be the densities and undisturbed thicknesses of the upper and lower layers. As the characteristic horizontal scale L^* we use the Rossby deformation radius $\lambda = [g(\rho_2 - \rho_1)H_1H_2/\rho_0 f_0^2 H]^{1/2}$, where g is the gravitational acceleration, ρ_0 and f_0 are the mean density and Coriolis parameter, and $H = H_1 + H_2$ is the total depth. Assuming g = 9.8 m/sec^2, $\rho_2 - \rho_1 = 10^{-3}$ g/cm^3, ρ_0 = 1 g·cm^{-3}, $f_0 = 10^{-4}$ s^{-1}, $H_1 = H_2$ = 500 m, we obtain λ = 15.65 km.

In the numerical experiments discussed below, the radii of all vortices are assumed to be initially equal to λ, and the maximum distance between the hetons was no more than an order of magnitude greater than λ. Under these conditions, we may use the f-plane approximation and local Cartesian coordinates. The initial state was specified in such a way that the position of the center $(x_c)^j_k$, $(y_c)^j_k$ of the k-th circular vortex in the j-th layer is given by the formulas

$$(x_c)^j_k = (-1)^{k-1}[l + (-1)^j A], \quad (y_c)^j_k = (-1)^{k-1}B; \quad j, k = 1, 2. \tag{1}$$

In the f-plane, all horizontal directions are equally significant, and the line $x = 0$ is chosen as an axis of antisymmetry only for convenience. The values of A and B in Eq. (1) are fixed (A = 1, B = 5), and l is the varied quantity ($|l|$ is half the distance between centers of pairs at the initial instant).

In the numerical calculations, we used a two-layer version of the contour dynamics method [3], based on the law of conservation of quasigeostrophic potential vorticity in each of the layers [7], according to which the potential vorticity W^j_k of each of four vortices, which can be assumed constant at the initial instant, is also constant over time. The condition of baroclinic compensation, which is the distinguishing characteristic of hetons, is in the present case $W^1_1 = -W^2_1 = -W^1_2 = W^2_2 \equiv W$. The value of W is calculated from the requirement that the dimensionless translational velocity of the hetons be equal to unity, under the assumption that in the initial stage of motion the pairs do not interact so that the distributed vortices may be replaced by point vortices. This gives W = -10.36. Assuming the characteristic velocity V^* = 1 cm/sec, we obtain the time scale $T^* = 15.65 \cdot 10^5$ sec = 18.11 days.

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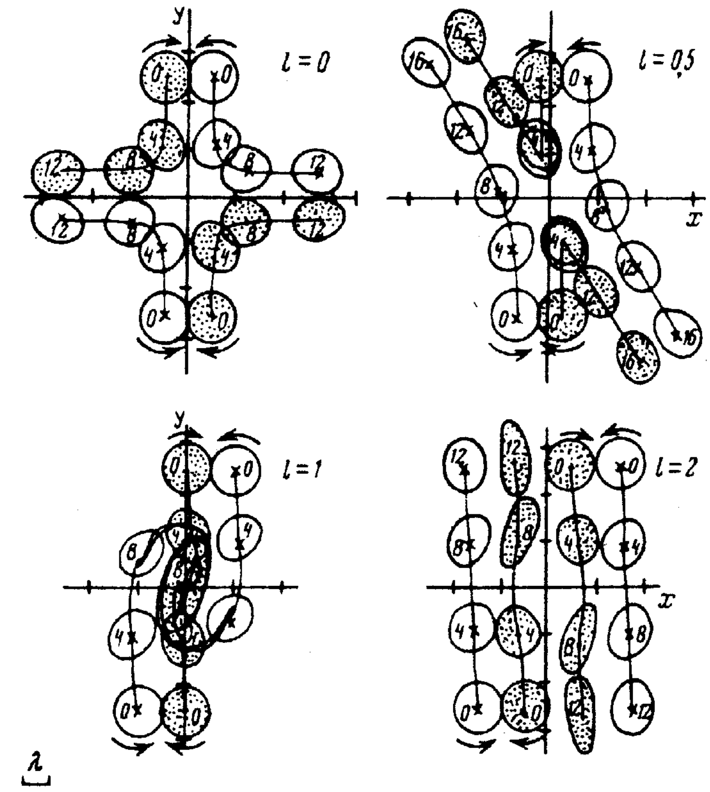

Fig. 1. Head-on collision of two distributed hetons at various values of l. Vortices in upper layer are stippled. Solid lines show paths of centers of vortices; markers show their positions at specific instants.

3. Numerical experiments for a variety of values of l had led to the identification of three types of interaction: I, change of partners; II, coalescence of the vortices in the upper layer; III, divergence of the hetons. In all three cases, the vortex pairs moved in nearly linear fashion in the first stage of motion, and the interaction between them became pronounced only when they approached to a distance of about 4.

Let us consider each of the three types of interaction separately.

Type I. As the hetons approach each other, the vortices in the upper and lower layers begin to move apart, then form new pairs (changing partners). The distance between the centers of the vortices in the new hetons is about 2, and these hetons move apart in opposite directions along a line whose angle α with the x axis increases with l from 0 in the case of a central collision ($l = 0$) to some value $\alpha^* > 0$. In our calculations, the maximum value $\alpha^* \approx 90^\circ$ was obtained at $l = 0.5625$.

As will be seen from Fig. 1, even at $l = 0.5$, the shapes of all four vortices are virtually unaltered by the interaction, and the paths of their centers are well described by the point vortex model. By increasing l in steps of $\Delta l = 1/64$, we found that when $l = 0.5 + \Delta l$, the pattern is qualitatively the same, but in the very next step the vortices in the upper layer coalesce, then separate again under the influence of the vortices in the lower layer, which continue to move in opposite directions. In this situation there is a partial exchange of mass between them. In the subsequent stage, the separated vortices become nearly circular, their paths turn through the appropriate angle, and they begin to move along straight lines together with the lower vortex of the second pair.

Type II. For the given step Δl, the transition to this type of motion occurs when $l = 0.5 + 6\Delta l = 0.59375$. In this situation the vortices of the upper layer coalesce and form a singly connected fluctuating vortex structure rotating around the origin. The vortices in the lower layer show practically no change in shape, and rotate around the same center in nearly circular orbits. The central vortex typically emits filamentary jets, which become more and more pronounced with increasing l.

Figure 2 shows calculation results indicating that the configuration resulting from this type of interaction is close to some (apparently stable) steady state. The initial arrangement of the vortices in this experiment is approximately equivalent to that at time $t = 8$ in the case $l = 1$ in Fig. 1, if we neglect the presence of the filaments emanating from vortices and assume that the area of the ellipse is equal to the sum of the areas of the originally circular anticyclonic vortices. The calculated motion of the system of vortices is shown for a time interval equal to approximately two periods of rotation. A similar calculation for the case $l = 1.75$ indicates that relatively rapid convolution of the vortex sheet begins on the periphery of the elliptical vortex, followed by its ejection after about 5 dimensionless time units. As a result, the elliptical vortex becomes more compact and continues to rotate in the same fashion, together with the circular cyclonic vortices. With increasing l,

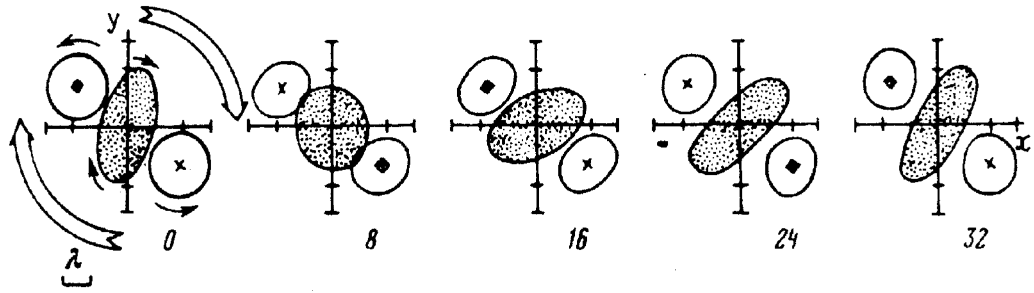

Fig. 2. Connected motion of system consisting of anticyclonic vortex in upper layer and two cyclonic vortices in lower layer.

there is an increase both in the radius of the circular orbits of the cyclonic vortices and in the period of rotation.

Type III. When l is increased further, the vortices in the upper layer, which coalesce as the hetons approach each other, rotate around the common center only enough to separate again and continue moving together with the corresponding cyclonic vortices in the lower layer. The boundary of the transition to type III may be tentatively assigned the value $l = 1.9375$. The partial coalescence of the vortices in the upper layer occurs even with $l = 2 - \Delta l$. At $l = 2$, the anticyclonic vortices move apart without coalescing, but with a considerable distortion of shape, a distortion which then becomes less pronounced with increasing l. Here, as in the first type of interaction, the paths of the centers are well described by point vortices.

4. The calculations indicated that in addition to the quite obvious types of interaction I and III, there is an intermediate type II, which occurs (under our choice of conditions) at least when the interval between the centers of the colliding vortices is between 1.2 and 3.8 deformation radii. We emphasize that this property is typical of the distributed hetons. Our calculations indicate that in the limiting case of single vortices, there occurs a conversion from type I to type III movement as l is increased, at which point $\alpha_* \approx 180°$. Case II represents a state of circular rotation of the entire system in which the cyclonic and anticyclonic vortices are arranged in pairs at opposite ends of diameters d_c and d_a that intersect at right angles. The quantities d_c and d_a are determined from the corresponding steady-state solutions of the equations of motion of point vortices [2]. This case $d_a < d_c$, represents one of the two branches of the solution; it is unstable and bifurcates to case III.

Thus, our numerical experiments confirm that in modeling a connected state of type II, the finality of the dimensions (non-point nature) of the vortices stabilizes the system, as occurs, for example, in the case of a Karman Street [8, sect. 159; 9].

5. In Fig. 1, $l > 0$ everywhere. A change in the sign of l results in mirror reflection of the figure in the (x, y) plane across the y axis. In this case, a type II condition will involve, among other things, cyclonic rotation of a quasi-elliptical vortex in the lower layer, with the anticyclonic vortices moving in circular orbits in the upper layer. Naturally, the symmetry property (as described) will occur only in the simple case $H_1 = H_2$ that we have described. Note that a two-layer vortex structure with noncoincident vortex centers and with opposite signs of the potential vorticity in the upper and lower vortices may, as a first approximation, serve as a model of a baroclinic vortex with an inclined axis and with rotation in opposite directions in the near-surface and near-bottom intervals. The problem of determining the approximate range of values of the parameters L^*/λ, $A, H_1/H_2$ in which connected states of type II can exist in a system of two such vortices is of interest.

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