Are there fundamental laws in hydrology?

S.G. Dobrovolski, V.P. Yushkov, T.Yu. Vyruchalkina, O.V. Sokolova

Institute of Water Problems of the Russian Academy of Sciences, Moscow, Russia

sgdo@bk.ru

Abstract

A global "power law -2" of long-term natural changes in total volume of the terrestrial water mass and of the natural eustatic changes in the global mean sea level has been proposed using coastal gauges data and satellite altimetry. A "power law -1/2" of the dependence of the coefficient of variation of the annual river discharge on the runoff depth was formulated based on analysis of more than 2000 time series of "natural" rivers' annual runoff. Additionally, a law of "statistical invariance" of the water exchange coefficient of drainage lakes vs. average runoff through the lakes was hypothesized using data from 249 lake systems of the world. The considered regularities were justified with the use of the climate system stochastic models concept by Klaus Hasselmann and for the reasons of dimensions of its hydrological objects (rivers, lakes, glaciers) participating in the processes of moisture accumulation and transfer on the land surface.

Keywords: laws in hydrology, variations of terrestrial water mass, coefficient of variation of the river runoff, coefficient of water exchange of drainage lakes.

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This is a theoretical paper and has no data to archive. Not applicable

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S.G. Dobrovolski: Conceptualization, Formal analysis, Writing - Original Draft; V.P. Yushkov: Formal analysis, Writing - Original Draft; T.Yu. Vyruchalkina: Investigation, Data Curation; O.V. Sokolova: Investigation, Data Curation.

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Introduction

In various scientific areas, first in mathematics, physics, chemistry, and biology, and then in the earth sciences (i.e., “power law -5/3” in meteorology or “power law -2” as suggested by Klaus Hasselnmann in climatology), laws of nature, “stated regularities in the relations or order of phenomena in the world…” (Britannica…, 2017) have been revealed. The mathematical formulation of these laws has, as a rule, a very simple form.

At the same time, hydrologists, until now, have not attempted to describe similar fundamental relationships between the main processes and objects on the land surface. "*The hydrology is considered more as technology, than as science... Although practical problems of regulation of waters gave birth to the hydrology, now they slow down its development*" (Vinogradov&Vinogradova, 2008, p. 304–305). However, the accumulation, in recent decades, of a considerable amount of information on hydrological objects and processes via numerical statistical analysis seemingly allows us to reveal inherent, quasi-universal laws. The present article represents such an attempt.

In the land hydrology it is not easy to use simple dynamic laws, because hydrological systems are not the continuous media. However, global general laws uniting all variety of forms of accumulation of moisture and water streams must be shown through the general conservation laws and their global balance.

The most important objects of the land hydrology – glaciers (including ice sheets), rivers and lakes have significantly different time scales of water accumulation and discharges. Besides, the land surface hydrological processes are under the influence of very large number of external and/or unknown factors. Thus, the laws which are presumably operating stochastic dynamics of the listed types of objects will inevitably have statistical character (just as "power law -5/3" in meteorology or "power law -2" in climatology). At the same time, the mentioned randomness in hydrology seemingly has the order, strict regularities and laws as well. The aim of the present work is to discuss these laws, that is to reveal an order in the chaos of the dimensions and local properties of hydrological systems.

Global scale. "Minus two power law" of natural variations of the terrestrial water mass

The most fundamental, integrating hydrological parameter is the total water mass on the land. This parameter generalizes properties of all terrestrial hydrological objects. In the previous works of one of our co-authors (i.e., Dobrovolski, 2000) the following hypothesis was proposed: long-term *natural* changes of the annual terrestrial water mass might be described by a model close to the discrete Wiener process with spectral density close to a straight line with tangent -2 in the bilogarithmic scale.

Direct instrumental assessment of long-term variations of the total terrestrial water mass is extremely difficult or even impossible currently. However, we can estimate changes in the above parameter by means of calculating the eustatic changes in the global mean sea level (EGMSL). In this case we accept the hypothesis of constancy of the global water mass based on the insignificant and nearly equal speed of juvenile waters' inflow and dispersion of molecules of water in space (World water balance…, 1974).

For example, we can estimate the EGMSL spectral density using early registrations of the mean sea level, generalizing observations at coastal gauges (Kliege, 1978; Fairbridge & Krebs, 1962; Gornitz et al, 1982; Barnett T.P., 1983). The results of the analysis of these time series confirmed the above "-2 power law" hypothesis. Unfortunately, later, less attention was paid to reconstructing EGMSL series based on coastal gauge data. However, there is a variant of the EGMSL series (Jevrejeva et al., 2014) (included into reports (IPCC, 2019:

IPCC Special…, 2019)). The results of this series' analysis were analogous to those described (Dobrovolski, 2000)[1].

To show that changes in the EGMSL were not "a measurements mistake", and were confirmed by other types of measurements, we will address (described in more details) analysis of the second, seemingly more adequate, variant of the EGMSL series, which was obtained using satellite data. There are several types of such estimates, we chose a series of monthly values of the EGMSL from the GRACE (IPCC, 2019) project with accurate removal of both the steric component and average seasonal dynamics. This series covers the period from the beginning of 2003 through the end of 2014 and is shown in Fig. 1. It is important to stress that the considered period (2003 – 2014) was relatively "calm" with respect to changes in the mean global temperature (GT): our estimation of the GT trend during this period demonstrates the rate of only 0.003°C/year, i.e., approximately 12 less than during the period of maximal heating period from 1976 through 1998 (0.035°C/year). Thus, time series in Fig. 1 can be considered as a "natural" EGMSL series. Moreover, as shown below, there is no statistically significant monotonous trend within this series.

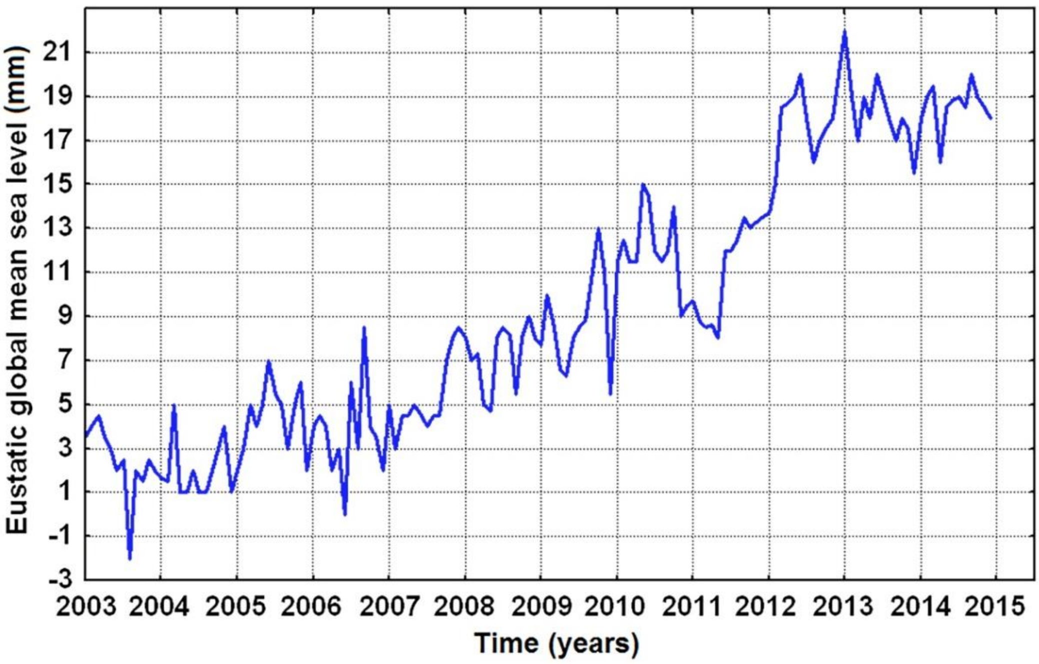

Fig.1 Eustatic GMSL monthly time series with both the steric component and average seasonal dynamics removed according to GRACE. Redrawn using data from (IPCC, 2019)

[1] For the results of the sea level fluctuations analysis at regional spatial scales and paleo temporal scales see, for example, (Yanko-Hombach & Kislov, 2017; Kislov, 2018).

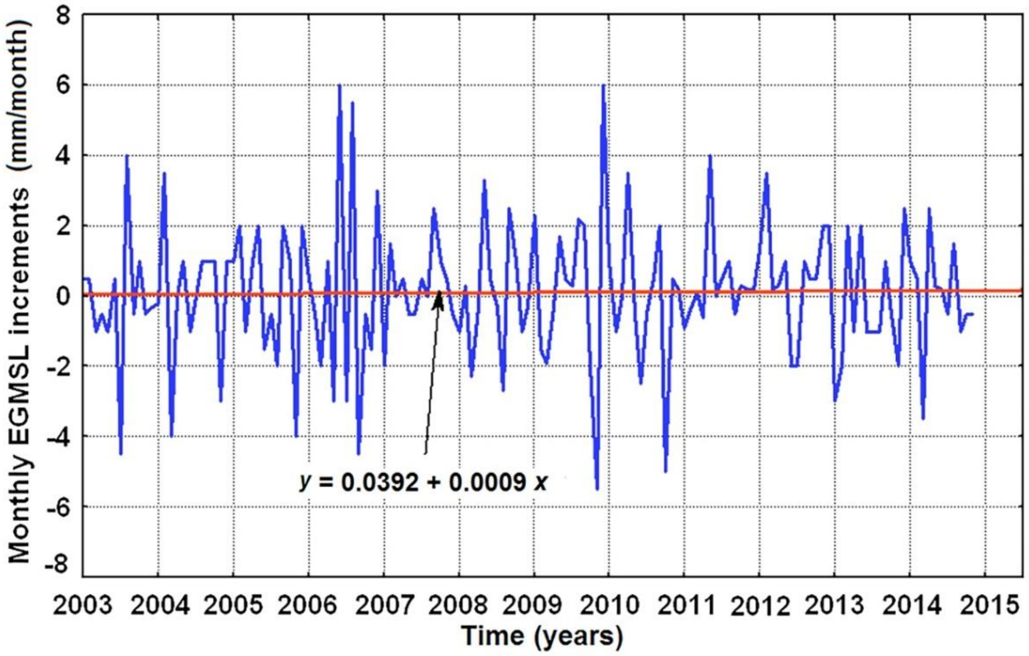

Fig. 2 Monthly increments of the series shown in Fig. 1 (the continuous line is a linear approximation)

It is evident, that for the existence of the statistically significant trend in Fig. 1, the average value of increments (Fig. 2) must be statistically different from zero (compared to the standard deviation of the monthly increments). For accurate analysis, we must also estimate the order of autoregression model of the observed series (Privalsky&Yushkov, 2018).

Methods for the statistical analysis of stochastic time series in geophysics have been well developed, including for the spectral analysis of short realizations (see for example Ulrych & Bishop, 1975 and Privalsky, 2021)). The first step in the analysis supposes the autoregression model *AR* of a finite order *M* (to be tested). The specific technique of such an analysis using the Maximum Entropy Method has been described in monographs (Dobrovolski, 2000; 2017).

To estimate the key parameter, the coefficient of correlation between the neighboring members of the time series (equal to the first trial coefficient of an autoregression model), we proposed a new formula based on a new method of generating Gaussian random values (using the so-called mirror-doubling generating algorithm). Thus, an amendment was proposed to the conventional sample estimate $r_{1,sam}$ (N is the series length):

$$\Delta r_{1,SAM} = \frac{0{,}945 + 3{,}05 r_{1,SAM}}{N} + \frac{4{,}73 + 18{,}6 r_{1,SAM} + 25{,}4 r_{1,SAM}^{2}}{N^{2}}$$

Calculations of the Akaike and Schwartz-Rissanen criteria for the assessment of *M* indicate the first order of process of autoregression for the description of the EGMSL series. The coefficient of autoregression is 0.94 when using both the scheme from Yule-Walker and the scheme from Burg. Thus, the coefficient of the first-order autoregressive model is extremely close to the unite value, and the model of the series approaches the discrete Wiener process model. Certainly, autoregression models do not consider explicitly all possible interactions in a hydrological system (evapotranspiration, runoff, transfer of water between latitudes). They only show spectral properties of a specific time series.

The analysis of the series of monthly EGMSL increments (Fig. 2) results in the following. The average value is very small at 0.101 mm/month, and coefficient of asymmetry is insignificant (-0.0493). The optimum order of the autoregression model estimated for the time series recalculated into the sample values of the Caussian random numbers was equal to zero.

In turn, a theoretical estimation of the mathematical expectation error (standard deviation divided by the square root of the length of the series) is 0.164 mm/month, i.e., 1.62 times more than the estimation of mean value. For comparison, the values of the analogous parameter for the time series of annual mean sea level increments, mentioned before (Kliege, 1978; Fairbridge & Krebs, 1962; Gornitz et al, 1982; Barnett T.P., 1983) were respectively 1.71; 2.17; 1.78; 1.27. Thus, that all 5 investigated variants of the annual and monthly EGMSL series do not demonstrate statistically significant monotonous deterministic trends.

Therefore, the time series of EGMSL increments can be described by the realization of the Gaussian white noise process with the mathematical expectation, which does not significantly differ from the zero value (the normal probability density is visible in the histogram in Fig. 3).

Finally, the obtained results indicate that *natural* eustatic changes of globally averaged values of the ocean level during the period of small changes in the global temperature are close to the realization of the discrete Wiener process, i.e., the random walk model. Respectively, spectral density of the land water mass fluctuations, estimated using an autoregression model, asymptotically approaches, in the bilogarithmic scale, a straight line with an angle tan of -2 (Fig. 4). This corresponds to the fundamental "power law -2" (Brownian noise) of the global natural fluctuations within the climate system described by the stochastic climate models theory by K. Hasselmann (Hasselmann, 1976)[2]. Physical bases of this theory for application to hydrological processes were generalized in (Dobrovolski, 2000).

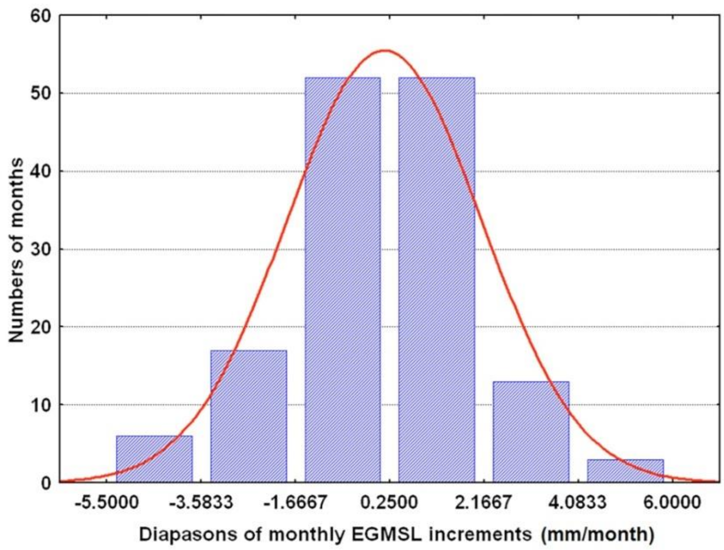

Fig. 3 Histogram of the probability density of monthly EGMSL increments (the continuous line – probability density of the Gaussian distribution)

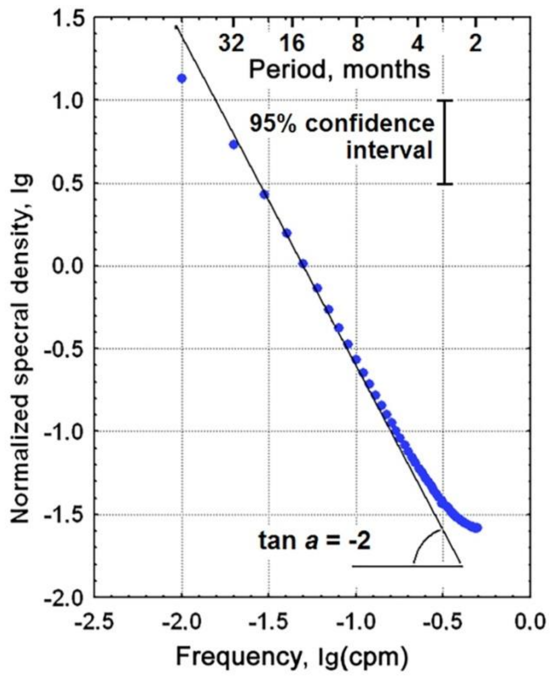

Fig. 4 Normalized spectral density ($S(f)/\sigma^2_a\Delta t$) of the EGMSL changes shown in Fig. 1. Vertical line denotes the 95% confidence interval. The straight line has an angle tan of -2

[2] Note that some researchers express doubts with respect to the range of applicability of Hasselmann "-2" law: see, for instance, (Lovejoy, 2019).

Mathematically, the “power law -2” follows from the equation of Burg connecting the spectral density of the time series with the autoregression process of the order *M*:

$$S(f) = \frac{2\sigma_a^2\ \Delta t}{\left|1-\sum_{j=1}^{M} c_j exp(-i2\pi f j\Delta t)\right|^2}, \qquad (1)$$

where *S* is the spectral density, *f* is the frequency (cycles per month), σ^2_a is the variance of residual white noise, Δt=1 (month), *j* is the number of the coefficient of autoregression, and c_j is the autoregression model coefficient. Apparently, for low enough frequencies, at *M*=1 and with coefficient c_1 approaching the unit value, the spectral density at the bilogarithmic scale asymptotically approaches a straight line with tan -2.

Dependence of coefficient of variation of the river runoff on the mean runoff depth: “power law -1/2”

The main process of the horizontal movement of moisture at the regional scale is river runoff. The most important and most informative indicator of year-to-year and long-time changes of a specific rivers’ runoff is coefficient of variation, C_V, i.e., the standard deviation of the annual runoff sequence divided by the average long-term annual river runoff. In (Dobrovolski, 2017) it was shown that the series of annual runoff values for “main” type of rivers (not lake rivers and not regulated by hydraulic engineering constructions) are generally close to realizations of stationary uncorrelated in time random sequences (white noise). Thus, in most cases C_V provides the main information on the long-term changes of a specific river’s runoff. The relationship between C_V and the average value of the parameter that forms the runoff, the annual runoff depth, is of a great interest.

In Fig. 5 a graph constructed using approximately 2000 annual runoff series for rivers of the “main type”, with data collected from 5 international archives, is shown. This joint archive is described in detail in (Dobrovolski, 2017).

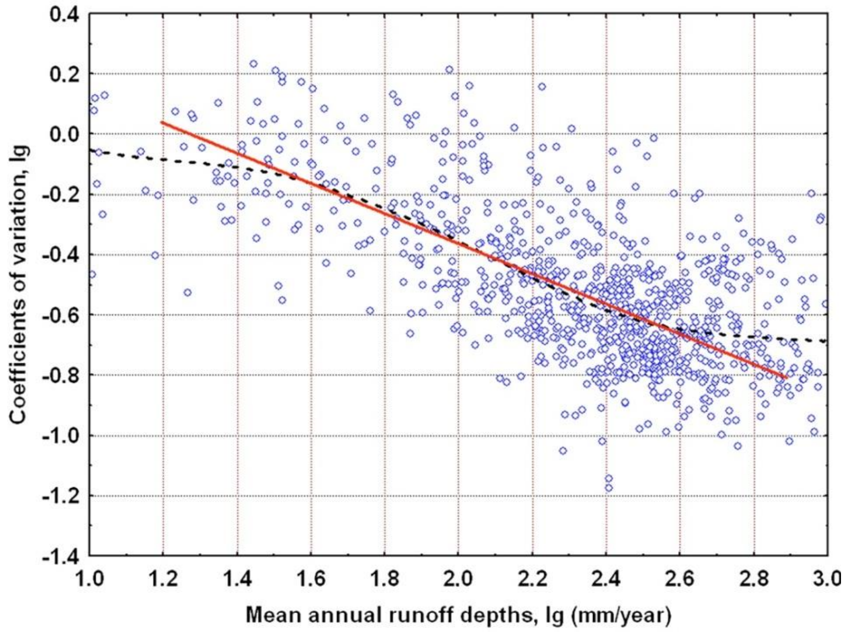

Fig. 5 Relationships between the coefficients of variation C_V of the annual river runoff and mean annual runoff depths in bilogarithmic scale. The dashed line is an approximation by the method of the spatially weighted least squares. The continuous line designates a straight line with an angle tan of -1/2

It is evident that each river/gauge demonstrates its own, random value of the runoff depth. In order to obtain a statistical law, it is necessary, first, to assess average coefficient of variation for rivers with similar river depths. In this way, one can obtain a "weighted" estimation of the relationship between the two parameters. The appropriate curve is shown by the dashed line in Fig. 5

Often, statistical laws arise in situations when characteristics of poorly correlated processes are summarized in time or in space. This allows, upon neglecting regional features of the runoff and difference in the areas of watersheds, as a first approximation, the following illustration and very simple model of the considered phenomenon.

The annual river depth is calculated by the formula:

$$D = Q/S, \tag{2}$$

where Q is the annual volume of the river discharge measured at a river gauge, and S is the area of the appropriate watershed.

Let us assume that a river depth during some "average" year can be presented as the sum of “elementary river depths” formed by a series of synoptic situations (atmospheric fronts and cyclones) that bring enough precipitation to create runoff:

$$D = d_1 + d_2 + \ldots + d_N, \quad (3)$$

where N is an average (for this gauge) number of the specific/elementary atmospheric situations during a year, which contributes to the general river depth. Let's assume now, for simplicity that standard deviations $\sigma(d_i)$ and mathematical expectations of the specific runoff depths’ portions $M(d_i)$ from each atmospheric situation with precipitation are equal to each other, that is, are constant in time and are equal to $\sigma(d)$ and $M(d)$. In addition, we neglect the seasonal dynamics, snow accumulation, and the asymmetry of the river depths’ probability density, i.e., we maximally simplify the task.

Let us assume also that values d_1, d_2, … are not correlated with each other in time. This hypothesis is confirmed by the small integral temporary scale (correlation time) of synoptic processes in the atmosphere (2-10 days). Data from water balance stations for a wide range of natural zones, where most rivers are located, confirm the absence of monthly river depths’ correlation in time (Dobrovolski, 2017); this may be an additional reason for adopting the above assumption. Thus, we can write:

$$\sigma(D) = \sigma(d) N^{1/2}. \quad (4)$$

Using formulas (2) and (4), we can express the annual coefficient of variation in the following form:

$$C_v = \sigma(D)\, S/Q = \sigma(d) N^{1/2}\, S/[M(d) N S]. \quad (5)$$

It is easy to show that

$$C_v = L/D^{1/2}, \quad (6)$$

where $L=\sigma(d)/[M(d)]^{1/2}$= const. for a specific river gauge. Note that the equation (6) can also be obtained using the method of dimensionality.

Fig. 5 shows that the straight line with tan -1/2 coincides with the approximating line within the main range of the annual river depths – from approximately 30 to 300 mm/year. This means that coefficient L is not only constant for the specific river gauge, but its global mean value (mathematical expectation) has the same order for the ensemble of different river gauges. If, within the mentioned above river depths' interval, we extract a straight line with tan -1/2 from the graph in Fig. 6, we obtain a field of points, which can be approximated by a horizontal straight line. Thus, parameter $\sigma(d)/[M(d)]^{1/2}$ seemingly is an invariant with respect to the annual river depths within large range of natural zones, except for regions with extremely high or extremely poor humidification.

For more droughty watersheds, the approximating curve in Fig. 5 is below the -1/2 line. Seemingly, this is because the number of days with precipitation in these natural zones is extremely small, and the statistical dependence on the number of synoptic situations causing runoff here is not significant (apparently, a more important role is played here by differences in intensity of very rare precipitation).

For extremely humid regions with river depths more than 300 mm/year, the approximating curve also departs from the straight line of "law-1/2". Perhaps, this is related to the extremely long (in certain cases seasonal) duration of the periods of precipitation and to the considerable correlation between neighboring "elementary" river depths. Furthermore, an assumption about the constancy of the $M(L)$ value is not evident here. In this case, our hypotheses, which are the basis for the above formulas, do not work.

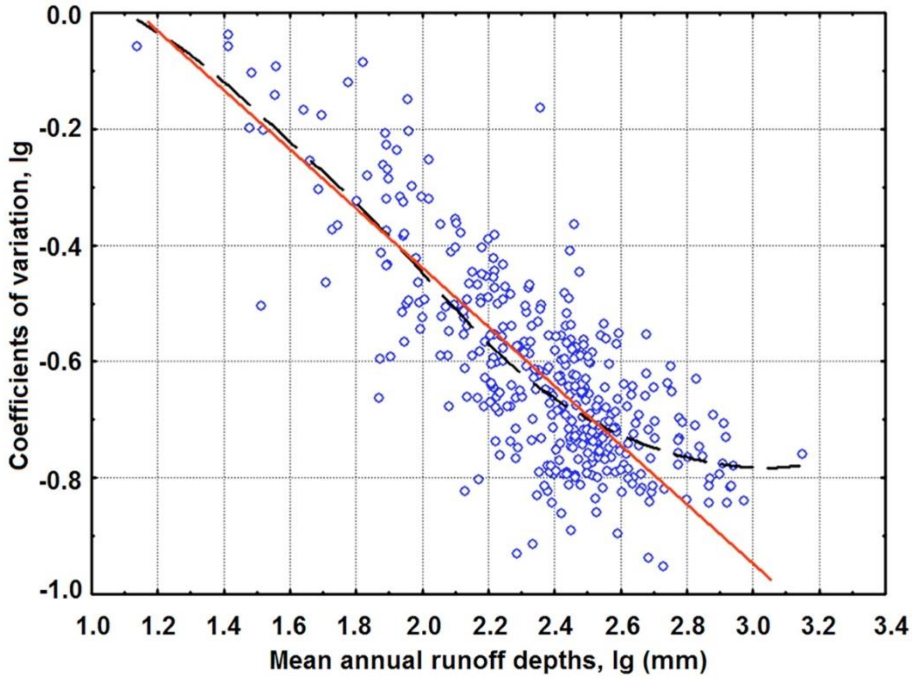

Fig. 6 Same as Fig. 6, but for river gauges within the territory of the Russian Federation. The dashed line is an approximation by the method of spatially weighted least squares; the straight line has an angle tan of -1/2

Thus, by means of extremely simplified calculations that consider the main processes, we can illustrate the “power law -1/2” of dependence of coefficients of variation on the river depths. Remarkably, manifestation of this law is characteristic not only at the global scale but also at the scale of macro-regions and of basins of the largest rivers. For example, the relationships between the annual coefficient of variation and the annual runoff depths for the territory of the Russian Federation are given in Fig. 6. Calculations were made with data for the "main", rivers using our classification type of not a lake and not regulated by large hydraulic engineering constructions.

Qualitatively, similar patterns are characteristic for the basins of rivers, such as the Ob, Yenisei, and Lena.

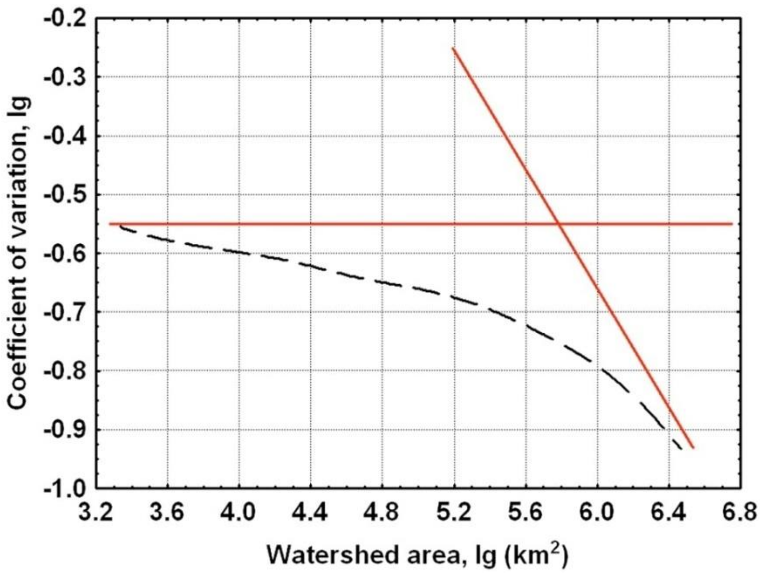

Fig. 7 Coefficient of variation of the annual runoff vs. the watershed area (both in logarithms); rivers of the world are of the main type

If the main reason for the "power law -1/2", for describing relationships between C_V and D, is the limited temporary correlation of synoptic processes that result in precipitation, the limitation of spatial correlation of these processes is the reason for a similar pattern for the coefficient of variation C_V and the area of the river basin S. However, the last regularity is less distinct because the number of river basins with areas that significantly exceed the characteristic dimensions of synoptic structures is small. Therefore, the line with an angle tan -1/2, in bilogarithmic scale (Fig. 7), looks more like an asymptote than a regression line. In the left part of the graph, where the size of the basins is much less than the characteristic size of synoptic structures, S does not influence the coefficient of variation, and the approximating line approaches a horizontal asymptote.

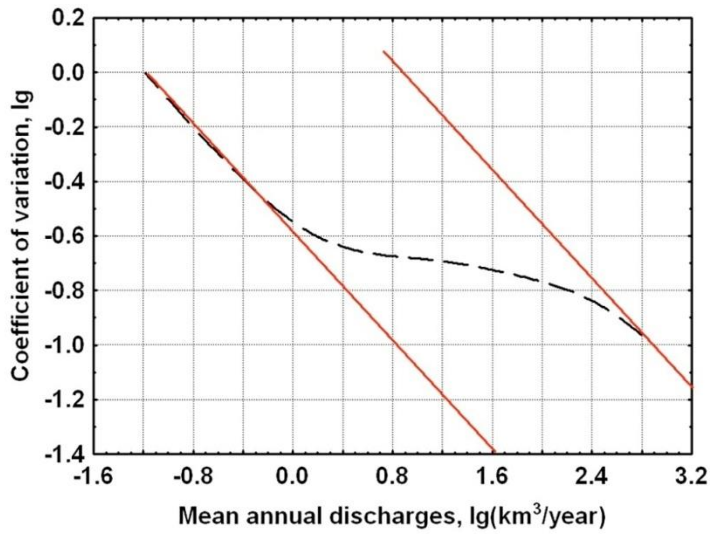

Fig. 8 Annual coefficients of variation vs. annual runoff discharges in logarithms. Russian rivers of the main type of feeding. The dashed line is the approximation by the method of spatially weighed least squares. Straight lines correspond to the “power law -1/2”

Both “power law -1/2”, with respect to the relationships between the coefficient of variation C_V and the runoff depth *D*, and the relationship between C_V and the area of the river basin *S* were used to make the graph “C_V vs. river discharge *Q*“ (Fig.8). Indeed, the annual runoff *Q* can be expressed as *D*S*. Within the diapason of low *Q*, where both *D* and *S* are small, the role of *S* is negligible, and the graph follows the pattern shown in Fig. 5. However, if discharges are extremely large, this can occur only in the case when *S* is great and the graph in Fig. 8 follows the pattern of Fig. 7. Finally, for the intermediate values of *Q*, i.e., for intermediate runoff depths and basin areas, a graph as in Fig. 8 approaches the horizontal asymptote, just like the right part of Fig. 5 and the left part of Fig. 7. Thus, a graph like Fig. 8 represents a combination of Fig. 5 and Fig.7. Here, we give an example of a graph C_V vs. *Q* for the territory of Russia because this is the biggest region of the world where data on runoff are homogeneous and obtained using the same methods.

Coefficients of water exchange for drainage lakes: global statistical invariance with respect to river discharge through lakes

Statistical generalization is capable of changing stereotypes based on elementary reasonings of "causality". The annual coefficient of water exchange of a lake, C_E is defined as the mean annual volume of water draining from a lake, *Q*, divided by the mean volume of the lake, *V*.

At first glance, it seems evident that the relationship between C_E and Q is a straight line with a tangent other than zero. However, calculations show that this idea is not confirmed with statistical generalization.

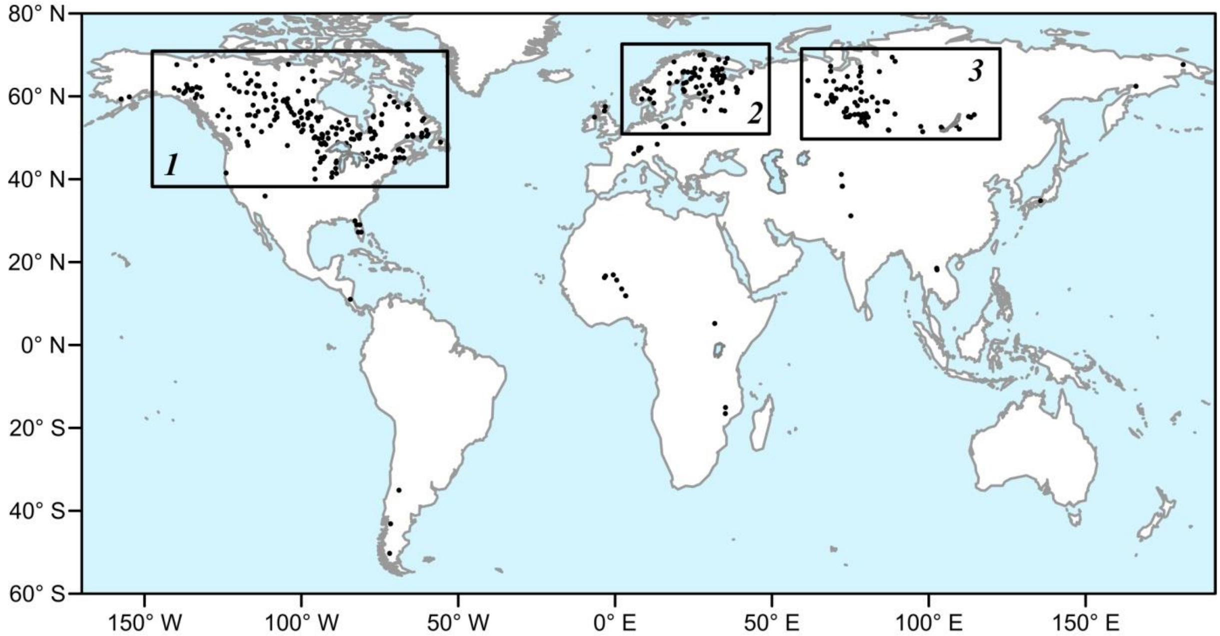

Fig. 9 River gauges on effluents used in this work. Figures designate the main areas of lakes: *1* –the North American area, *2* – North European area, *3* – North Asian area

The authors used information on drainage lakes of the world for which long (not less than 20 years) time series of discharges through the lakes were available. Total number of these lakes was 249. Data on the parameters of the lakes were taken from prior studies (Lehner & Doll, 2004; Messager et al., 2016; Rivers and lakes…, 1971; Ryanzhin & Geller, 2006; Singh et al., 2006; Starosolszky, 1974; https://wldb.ilec.or.jp; https://lakepedia.com; https://smhi.se; https://nve.no; https://nrcan.gc.ca); information on the runoff at river gauges close to the appropriate lake was taken from Appendix 1 of a monograph (Dobrovolski, 2017). The locations of lakes, specifically, the coordinates of sources of the respective lake rivers, are shown in Fig. 9. Drainage lakes are an overwhelming majority of all lakes in the world. Analyzed lakes of glacial origin (drift-dammed and drift-dammed-tectonic are the most numerous) – they make up 60.2% of the lakes considered in this article. "Tectonic" lakes (both tectonic and volcanic) make up 9.7%, and other types make up 30.1%.
The scatter point graph in Fig. 10a shows the relationships between the coefficient of water exchange C_E and runoff from the lake Q for the lakes of glacial origin (most numerous).

Unexpectedly, Fig. 10a demonstrates the absence of statistically significant connections between C_E and Q. Moreover, correlation coefficient between C_E and Q in Fig. 10a is only 0.02.

On the contrary, the graph of C_E and V (Fig. 10b) shows a strict relationship between the two parameters, which is close to linear in the bilogarithmic scale with a non-zero tangent of angle.

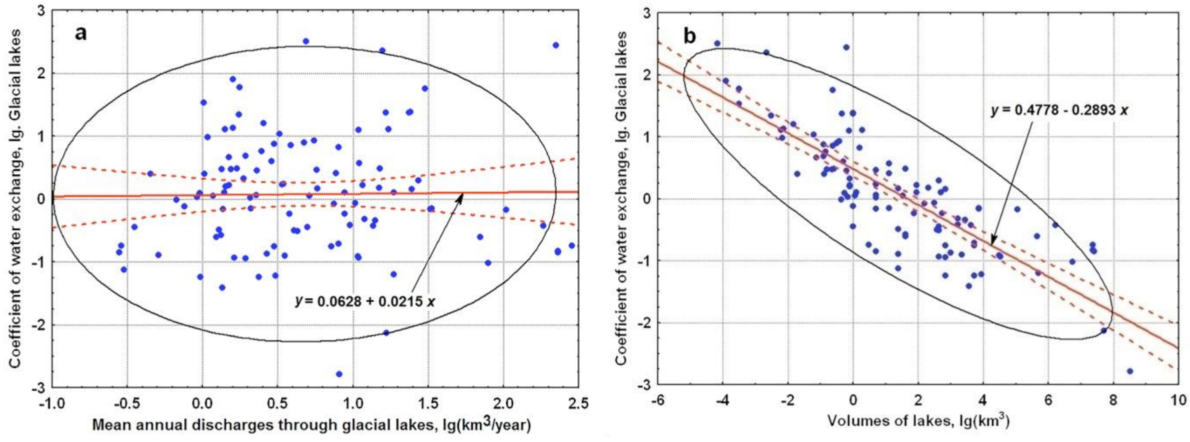

Fig. 10 Relationships between C_E and Q (a) and between K_E and Q (b), in the bilogarithmic scale. Straight lines are linear approximations by the method of spatially weighted least squares. Dashed lines designate 95% confidence intervals for the lines of regression, and 95% ellipses of dispersion are shown

The same results were obtained with respect to the analysis of C_E and Q of the tectonic lakes: the angle tangent of the regression line in this case was only 0.045.

Interestingly, the analysis of the data for each of the 3 areas in Fig. 9 gave similar results.

One possible explanation to the above "invariance" effect is the much larger statistical variety of volumes of lakes in comparison with the runoff from these lakes. Thus, the range of the runoff values under consideration is from 0.27 through 286 km^3/year (approximately 3 orders of magnitude) while the range of volumes of the studied lakes was from 0.0155 through 5000 km^3 (approximately 5.5 orders of magnitude). Thus, the range of volumes of lakes exceeds, approximately by 170 times, the range of variations of the annual volumes of the runoff. In other words, the volume of a lake has more opportunities to affect the water exchange coefficient than the runoff. Seemingly, a dramatic difference in dimensions of the key objects forming the basic parameters might be the fundamental reason for the considered

phenomenon: a watershed forming the runoff is significantly two-dimensional, while the kettle of a lake determining the volume of the lake water is essentially three-dimensional.
Note that, at a qualitative level, a similar idea concerning the discussed phenomenon was first expressed by K.K. Edelstein previously (Edelstein, 2005).

Conclusion

As in some other earth sciences, it is possible to call general statistical regularities that characterize key land water objects (glaciers, rivers, lakes) as laws ("laws of nature"). Briefly we will repeat the regularities found in this study.

- "Power law -2" of *natural* changes in the total terrestrial water mass at climatic temporal scales has been hypothesized in this paper. Informative indicators of such changes, assuming a constancy of the global water mass, are eustatic changes in the global mean sea level. This analysis, with the help of both measurements at coastal sea level gauges, and satellite altimetry leads to the description of the considered variations in the form of the discrete Wiener process. In turn, the generalized spectral density of the latter, in the bilogarithmic scale, is close to a straight line with the angle tangent of -2. The reason for this pattern may be atmospheric (synoptic) forcing of the underlying media without feedback, according to the general stochastic theory of climate models developed by K. Hasselmann.
- "Power law -1/2" of the relationship between the key parameter of long-term changes in river runoff, coefficient of variation C_V, and the annual runoff depth D, has been proposed. The analysis of this regularity using data from river gauges (for the "main type" of rivers – not lake, not regulated by hydraulic engineering constructions) leads, in the bilogarithmic scale, to a linear relationship between C_V vs. D with an angle tan of -1/2 for most runoff depth ranges. The limited *time* scale of the atmospheric (synoptic) structures, which are "sources" of the river runoff, might be the main reason for this phenomenon. The similar dependence, which is less accurately expressed, is the relationship between the coefficient of variation and the area of the river basin. In this case, the reason is the limited *spatial* scale of synoptic structures.
- One paradox is the statistical law of global invariance of the drainage lakes' coefficient of water exchange with respect to the average river runoff discharges through a lake. This

phenomenon can be explained by more variety in the volumes of lakes in comparison with the discharges through the lakes. In turn, the contrast between the variance of the runoff and volumes of lakes is caused by the difference in dimensions of the two types of hydrological objects: significantly two-dimensional river basin forming the runoff and the three-dimensional hollow of the lake forming its volume.

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Figure captions

Fig. 1 Eustatic GMSL monthly time series with both the steric component and average seasonal dynamics removed according to GRACE. Redrawn using data from (IPCC, 2019)

Fig. 2 Monthly increments of the series shown in Fig. 2 (the continuous line is a linear approximation)

Fig. 3 Histogram of the probability density of monthly EGMSL increments (the continuous line – probability density of the Gaussian distribution)

Fig. 4 Normalized spectral density ($S(f)/\sigma^2_a\Delta t$) of the EGMSL changes shown in Fig. 1. Vertical line denotes the 95% confidence interval. The straight line has an angle tan of -2

Fig. 5 Relationships between the coefficients of variation C_V of the annual river runoff and mean annual runoff depths in bilogarithmic scale. The dashed line is an approximation by the method of the spatially weighted least squares. The continuous line designates a straight line with an angle tan of -1/2

Fig. 6 Same as Fig. 6, but for river gauges within the territory of the Russian Federation. The dashed line is an approximation by the method of spatially weighted least squares; the straight line has an angle tan of -1/2

Fig. 7 Coefficient of variation of the annual runoff vs. the watershed area (both in logarithms); rivers of the world are of the main type

Fig. 8 Annual coefficients of variation vs. annual runoff discharges in logarithms. Russian rivers of the main type of feeding. The dashed line is the approximation by the method of spatially weighed least squares. Straight lines correspond to the “power law -1/2”

Fig. 9 River gauges on effluents used in this work. Figures designate the main areas of lakes: *1* –the North American area, *2* – North European area, *3* – North Asian area

Fig. 10 Relationships between C_E and Q (a) and between K_E and Q (b), in the bilogarithmic scale. Straight lines are linear approximations by the method of spatially weighted least squares. Dashed lines designate 95% confidence intervals for the lines of regression, and 95% ellipses of dispersion are shown