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Dynamic-stochastic models of rainfall and snowmelt runoff formation

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Abstract Possibilities for the development of dynamic-stochastic models of runoff formation with random inputs are discussed. Two models are described: the first allows the calculation of the statistical distribution of the maximum discharges of rainfall floods, and the second the statistical distribution of snowmelt flood volumes. Meteorological inputs are generated by the Monte-Carlo method. Physically-based models are used for the transformation of input data into runoff. The various models are applied to observation data from two watersheds.

Modèles dynamico-stochastiques de formation de l'écoulement pluvial et de l'écoulement de fonte des nieges

Résumé On considéré les possibilités d'élaboration des modèles dynamico-stochastiques avec des entrées au hasard. Deux modèles sont décrits: l'un permet de calculer la distribution statistique des débits maximaux des crues pluviales, l'autre, la distribution statistique des volumes de l'écoulement des eaux de fonte de neige. Des valeurs des facteurs météorologiques de l'écoulement sont générées par la méthode de Monte-Carlo. On emploie des modèles dynamiques partant d'une base physique pour la transformation des valeurs d'entrée. L'essai des modèles est effectué en partant des données des observations sur deux bassins versants.

INTRODUCTION

In many cases observed runoff data are too scarce or nonhomogeneous to determine flood frequencies with sufficient reliability using statistical analysis of discharge series. The nonhomogeneity of hydrological series is continuously increasing as a result of man-induced changes in river basins. Changes in land use, reclamation, reforestation and especially urbanization affect appreciably the conditions of runoff formation which, in turn, can have an impact on runoff characteristics. Such changes are most evident on small watersheds, but over long periods they are also observed on large river basins. Thus the necessity becomes more urgent for the development of methods of estimating the statistical characteristics for short runoff series or for man-induced changes of factors affecting runoff.

If runoff series are not sufficiently long but there are long-term measurements of meteorological factors, one can try to estimate the statistical characteristics of runoff using meteorological series as inputs to a runoff model, thus transforming meteorological data into runoff.

Velikanov (1949) seems to have made one of the first attempts to use that approach, determining the probability of the logarithm of maximum snowmelt flood discharge as a product of the probabilities of the logarithms of maximum snow equivalent and the duration of snow melting. Kalinin & Darman (1953) and Parshin (1953) applied this approach to the calculation of the statistical characteristics of snowmelt flood volumes using simple empirical relationships between flood volumes and meteorological factors. The series required for the determination of those characteristics were obtained by consideration of possible combinations of observed values of meteorological factors.

Eagleson (1972) was the first to develop a dynamic-stochastic model, using a physically based model for the description of runoff formation processes. Assuming that the duration and the average intensity of effective precipitation are distributed exponentially and that overland flow is described by the kinematic wave equations, Eagleson obtained analytical formulae which allowed one to calculate the statistical characteristics of the maximum runoff due to rainfall from the statistical characteristics of the rainfall and the hydraulic parameters of the watershed. This method ensured the determination of reliable statistical runoff characteristics not only for the case of short discharges series, but it also allowed, to some extent, for human impact on river basins to be taken into account.

The development of Eagleson's method and its application to practical tasks of calculating maximum discharges due to rainfall are described by Leclerc & Schaake (1972), Wood & Harley (1975) and Chan & Bras (1979). Hebson & Wood (1982) used the geomorphological unit hydrograph method instead of the kinematic wave equations in the description of the transformation of precipitation to runoff. Diaz-Granados *et al.* (1984) included the process of infiltration in their model.

A method similar to the one suggested by Eagleson (1972) was also realized by Carlson & Fox (1976) for determining the distribution of maximum snowmelt runoff (the snowmelt duration and average intensity were assigned as input functions, instead of the duration and intensity of precipitation).

In a series of papers, Eagleson (1978) used a sophisticated physically-based rainfall/runoff model describing in detail the processes of evapotranspiration and water absorption by soil. He obtained analytical relationships which expressed the statistical characteristics of water balance components and maximum discharges, both for separate floods and for the whole year, as functions of the statistical characteristics of precipitation and the physical parameters of river catchments, soil and vegetation. The use of analytical methods, however, necessitated the introduction of certain assumptions that significantly limited the application of those relationships.

The simulation of meteorological impacts on a river basin using Monte-Carlo methods and numerical methods of solving differential equations describing dynamic processes in hydrological systems allow one to increase the potential of dynamic-stochastic models and to extend the range of conditions for which they can be applied.

The models described below and the results presented are based on such an approach to the determination of the exceedance probabilities of maximum rainfall floods and snowmelt runoff volumes.

DYNAMIC-STOCHASTIC MODEL OF RAINFALL FLOOD FORMATION

The first version of this model was described by Kuchment *et al.* (1983). Later, some of the model components were improved and the model was applied to a new basin, the Golovesnia River (catchment area 29.5 km^2), located within the Dniepez River basin.

The Golovesnia basin was schematized as a series of four rectangular reaches, each with the same slope and soil characteristics, located along the main channel.

To calculate the channel flow, the kinematic wave equations were used:

$$\left.\begin{aligned} &\frac{\partial(h_r B)}{\partial t} + \frac{\partial(q_r B)}{\partial x} = q_1 \\ &q_r = n_r^{-1}\, i_r^{1/2}\, h_r^{5/3} \end{aligned}\right\} \qquad (1)$$

where h_r is the flow depth; q_r is the discharge through unit width of channel; width B is the total channel width; i_r is the slope of the channel bed; n_r is the Manning roughness coefficient; and q_1 is the overland lateral inflow rate per unit channel length. The width of the river was assumed to be constant along its whole length (B = 30 m from field data).

Numerical integration of equations (1) was carried out using an implicit three-point finite difference scheme with the Newton-Raphson method of solution of difference equations using a length step of 100 m and time step of 30 min.

The following equations were solved to determine the overland flow for each of the schematized rectangular reaches:

$$\left.\begin{aligned} &L\frac{dh_1}{dt} = (R - I)L - q_1 \\ &h_1 = 0.625\,(q_1\, n_1/i_1^{1/2})^{3/5} \end{aligned}\right\} \qquad (2)$$

where h_1 is the average depth of the overland flow; L is the length of overland flow to the river; R is the precipitation rate; I is the infiltration rate; i_1 is the slope of the overland flow; and n_1 is the Manning coefficient.

Infiltration was determined using the equation of soil moisture diffusion:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial Z}\left[D(\theta)\frac{\partial \theta}{\partial Z} - K(\theta)\right] \tag{3}$$

where θ is the volumetric moisture content of the soil, $D(\theta)$ is the moisture diffusion coefficient, and $K(\theta)$ is the hydraulic conductivity.

The following expressions were assigned as upper boundary conditions:

$$\left.\begin{array}{ll} R - E = -D\dfrac{\partial \theta}{\partial Z} + K & I > R - E \\ \theta(0,t) = \theta_s & I \leqslant R - E \end{array}\right\} \tag{4}$$

where θ_s is a constant (during rainfall θ_s was taken to be equal to θ_{max}); and E is the evaporation rate.

At the lower boundary, the moisture flux was taken to be equal to the hydraulic conductivity.

Evaporation, hydraulic conductivity and moisture diffusion were determined as:

$$\left.\begin{array}{l} E = k_e \cdot d_a(t) \cdot \theta(0,t) \\ K(\theta) = K_s \cdot (\theta/P)^{(2n+1.6)} \\ D(\theta) = K_s \cdot \psi \cdot n \cdot \gamma^n \cdot \theta^{(n+0.6)}/P^{(2n+1.6)} \\ n = 2.7 - 5.2 \times 10^{-6}\ \phi^{3.5}/\log_{10}(\gamma/\phi) \end{array}\right\} \tag{5}$$

where k_e is an empirical coefficient; d_a is the air humidity deficit; K_s is the saturated hydraulic conductivity; P is the volumetric porosity of the soil; γ is the maximum hydroscopicity of the soil; ϕ is the field capacity (% of the soil dry weight); and ψ is the matrix potential if the soil moisture equals γ.

An implicit four-point finite difference scheme with the solution of the difference equations by the double sweep method was used for the numerical integration of equation (3). The depth step was 100 mm; the time step was 1 day for a dry period and 30 min during rainfall.

Most of the parameters were determined from observed data. The average values of K_s and k_e were found using soil moisture content measured data; n_1 and n_r values were fitted by comparing calculated and observed runoff hydrographs for five rainfall floods. The values of all model parameters are shown in Table 1. Satisfactory agreements between observed and calculated hydrographs were obtained for five other floods (Fig. 1); the error in calculating the maximum discharge was less than 11% for all cases.

Measured data for eight years were used to develop a stochastic model of the meteorological inputs. Analysis of the data for the basin under consideration showed that the average duration of dry periods, $\bar{t}_d$, in the summer season is considerably larger than the average duration of rainfall, $\bar{t}_p$ ($\bar{t}_p$ = 2.9 h, $\bar{t}_d$ = 3.2 days). At the same time the dependence between

Table 1 *The values of parameters of the model for rainfall flood formation*

Parameter	*Value*
n_r	$5.0 \times 10^{-4}\ m^{-1/3}\ s$
n_1	$2.0 \times 10^{-3}\ m^{-1/3}\ s$
K_s	$3.5 \times 10^{-6}\ m\ s^{-1}$
θ_{max}	0.55
P	0.55
γ	0.07
ϕ	0.14
ψ	-550 m
K_e	$3.5 \times 10^{-8}\ m\ s^{-1}\ mb$

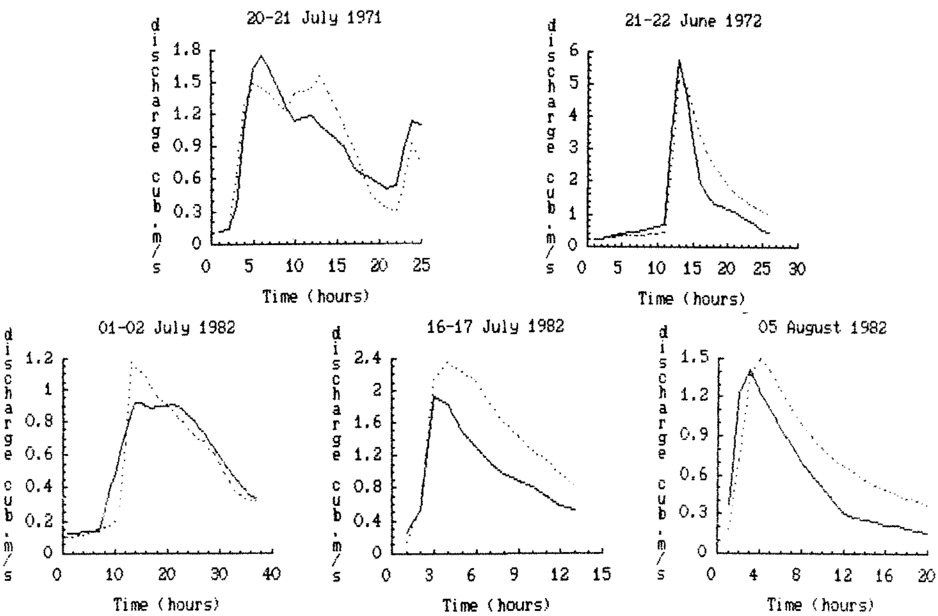

Fig. 1 *Comparison of observed (—) and calculated (.....) rainfall floods (Golovesnia River).*

adjacent values is not clearly expressed (coefficient of correlation, r_d = 0.22). The statistical distribution of t_d turned out to be close to an exponential one. A gamma distribution gave the best result for fitting the statistical distribution of rainfall durations (coefficient of variation 1.30). The same distribution was used to fit the empirical distribution of rainfall depth, h_a, and the average of air humidity deficit for the dry period d_a ($\bar{h}_a$ = 8.5 mm, C_v^h = 0.88; $\bar{d}_a$ = 6.7 mb, C_v^d = 0.35). The comparison of the empirical and fitted distributions of the input variables is shown in Figs 2–5. The coefficient of correlation between precipitation amount and rainfall duration was 0.71, and 0.72 between the average air humidity deficit for the dry period and the duration of this period. Analysis of the data on precipitation rate showed that

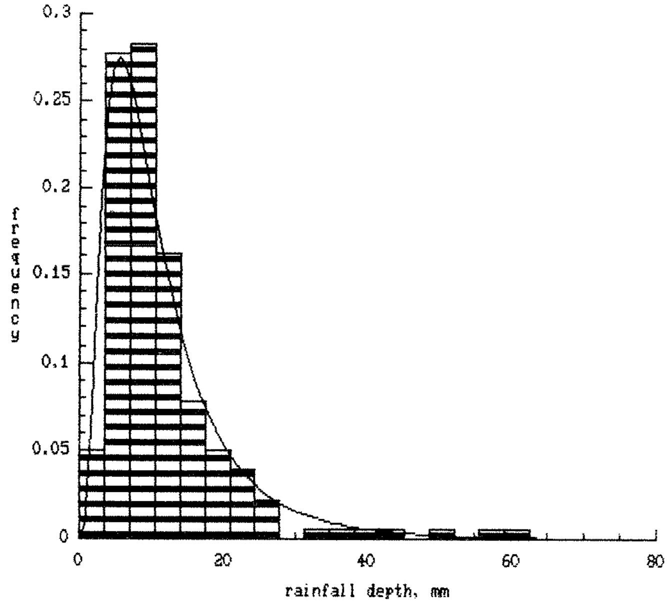

Fig. 2 Histogram of observed rainfall depth values fitted by a gamma distribution.

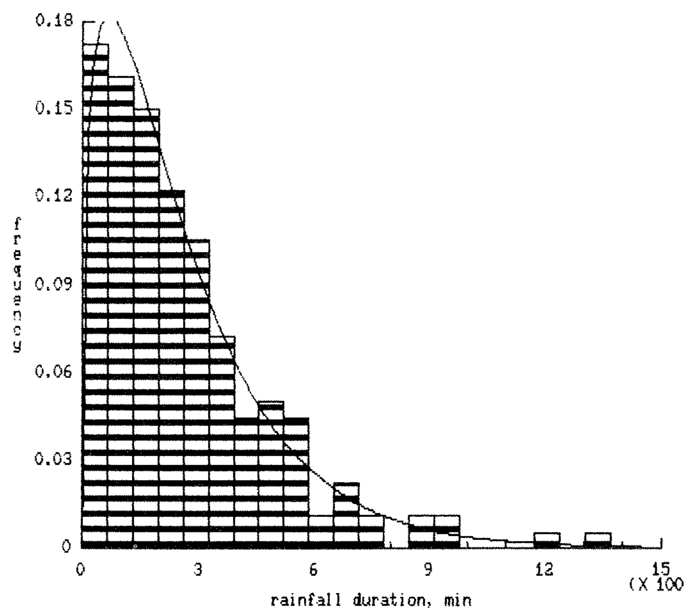

Fig. 3 Histogram of observed rainfall duration values fitted by a gamma distribution.

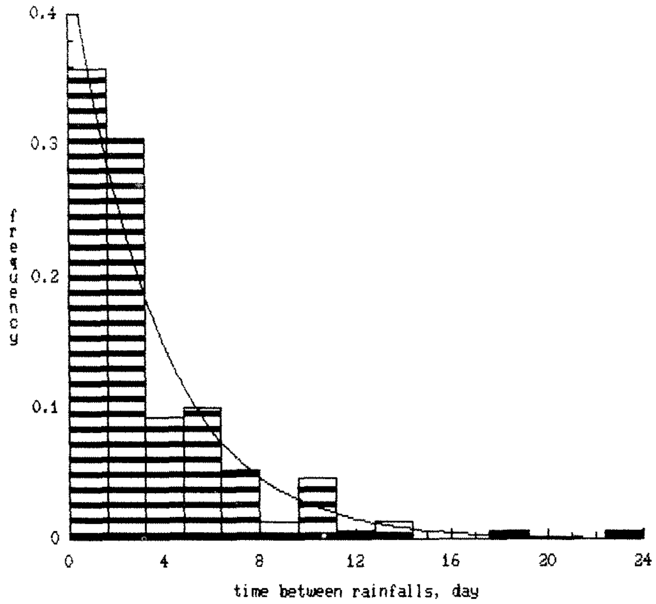

Fig. 4 *Histogram of values of time between rainfalls fitted by an exponential distribution.*

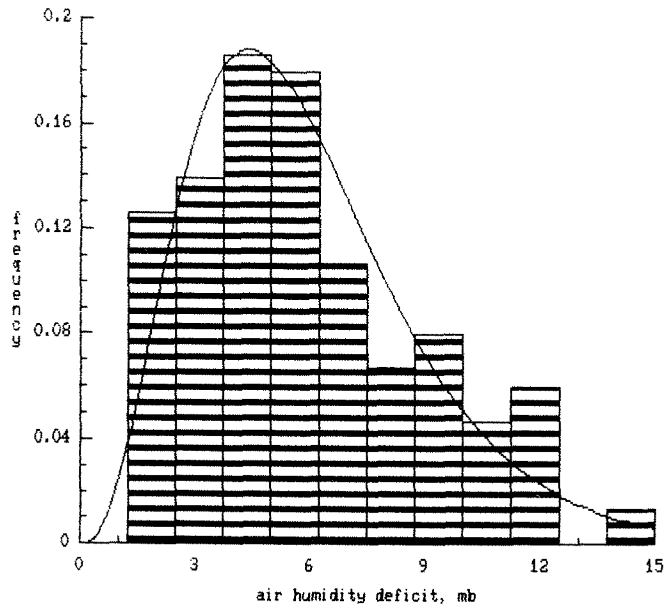

Fig. 5 *Histogram of average air humidity deficit values fitted by a gamma distribution.*

the typical hyetograph of summer rainfall can be presented as an isosceles triangle.

The generation of input data was realized by the following scheme. First, random generations of dry period and rainfall duration series for one season were carried out in accordance with the distribution of t_d and t_p (the sum of t_d and t_p was not supposed to exceed 120 days for any one season). The corresponding d_a and h_a values were then determined using regression equations with a stochastic component. The latter was assumed to be Gaussian white noise with expectation 0 and standard deviation equal to $\sigma(1 + r^2)^{1/2}$ (σ is the standard deviation of the value to be determined, and r is the coefficient of correlation between the independent variable and the variable to be determined). In spite of the fact that the distributions of the observed d_a and h_a values are close to gamma distributions, the adopted algorithm reproduces well the empirical distributions with the exception of the domain of great exceedance probability where there can appear small negative values. The comparison of histograms calculated using observed and generated h_a is shown in Fig. 6.

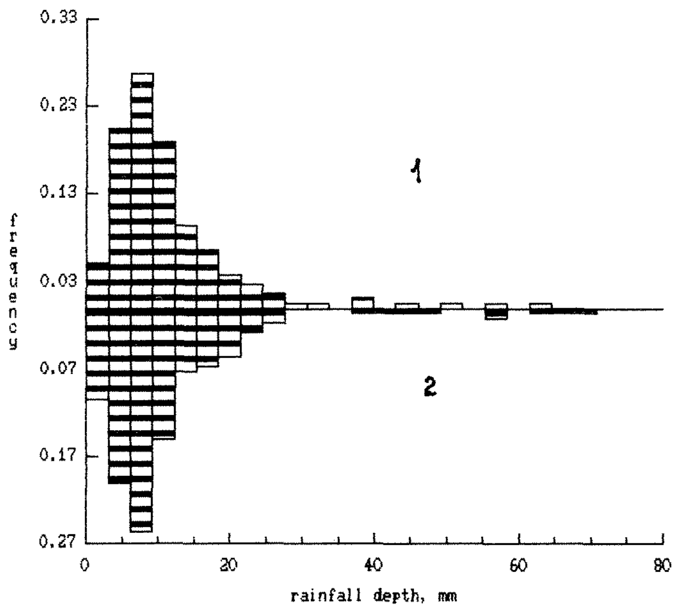

Fig. 6 *Comparison of histograms computed for (1) generated rainfall depths and (2) observed depths.*

Generated series of random values of t_d and d_a and hyetographs, with corresponding generated series of t_p and h_a values, were used as input data for the dynamic model. It was assumed that every season started from a dry period and that the moisture content of the upper 1 m soil layer was equal to the field capacity. For every dry period the change of soil moisture content was calculated. For every rainfall the losses to infiltration and the runoff hydrograph were calculated, and the maximum discharges were determined. The comparison of the statistical characteristics of the maximum discharges, obtained both with and without taking into account correlation between the input values, for 90 seasons (175 floods) shows that the neglect of correlation between t_p and h_a as well as between t_d and d_a can entail considerable errors (it resulted in a two-fold increase in the coefficient of variation of maximum discharge).

Runoff hydrographs for 380 seasons (541 floods) were simulated using the dynamic-stochastic rainfall/runoff model of the Golovesnia River, taking into account correlation between input values. The exceedance probabilities of the maximum discharge, calculated from these hydrographs were compared to those of the measured maximum discharges obtained on the basis of runoff observations during 50 floods. As can be seen from Fig. 7 there is a satisfactory agreement between the observed and calculated exceedance probabilities.

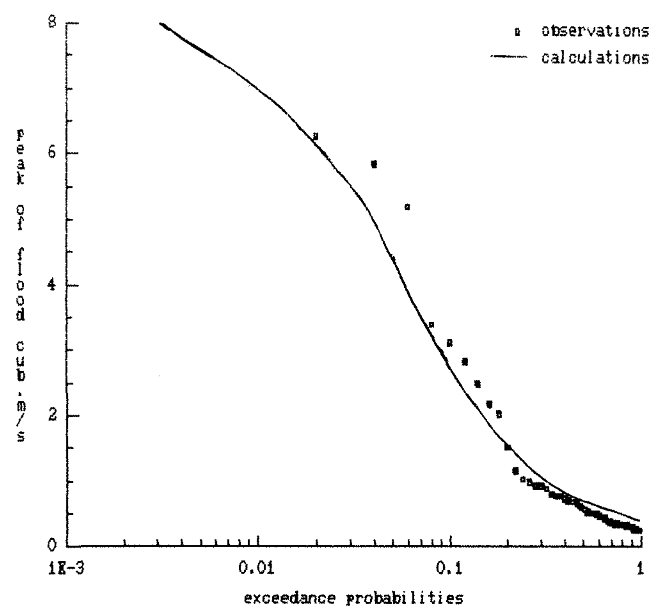

Fig. 7 Comparison of the exceedance probabilities of the maximum flood discharges calculated according to the rainfall/runoff model and determined using long-term runoff observations (Golovesnia River).

DYNAMIC-STOCHASTIC MODEL OF SNOWMELT RUNOFF FORMATION

Unlike rainfall floods, snowmelt floods have a long period of formation during which appreciable changes in meteorological processes can occur. Time-based statistical characteristics of precipitation and air temperature regime during soil freezing, snow accumulation and melting are studied less widely than rainfall characteristics. Such a situation restricts the possibilities of developing stochastic-dynamic models of snowmelt runoff with random inputs, in particular models enabling the calculation of a maximum discharge statistical distribution. The development of models allowing an estimation of the statistical distribution of runoff volumes during the snowmelt season on the basis of meteorological data seems more realistic.

A physically-based model of snowmelt runoff formation, with simplifications allowing its use in combination with a Monte-Carlo method for the calculation of the probabilities of runoff volumes, is presented below.

Soil moisture content, calculated prior to soil freezing using precipitation and air humidity deficit data during the whole summer–autumn period, was done in a similar way to the calculation of soil moisture transfer in the description of rainfall flood formation (equations (3)–(5)).

Taking into account that time intervals are long enough (about 1 day), the freezing front movement can be calculated as:

$$\lambda_f \frac{T_0}{\eta} - q_u = \mathfrak{H}\, \rho_w(\theta_+ - \theta_0)\, \frac{\partial \eta}{\partial t} \tag{6}$$

where $\mathfrak{H}$ is the latent heat of fusion of ice; ρ_w is the water density; θ_+ is the volumetric content of liquid water in the soil at the freezing front in the unfrozen part of the soil; θ_0 is the volumetric content of liquid water in the soil at a temperature near 0°C (assumed to be equal to the wilting point); η is the freezing depth; q_u is the heat flux from the unfrozen soil to the freezing front; T_0 is the temperature at the soil surface; and λ_f is the coefficient of thermal conductivity of frozen soil.

If it is assumed that, at sufficiently large depth, temperature has a constant value throughout the whole freezing period, the heat flux from the unfrozen soil to the freezing front can be approximately calculated as:

$$q_u = T_u(\lambda_u C_u/\pi t)^{1/2} \tag{7}$$

where C_u and λ_u are the heat capacity and the coefficient of thermal conductivity of unfrozen soil, respectively, and t is the time since the beginning of calculations. (Temperature at large depths of soil is taken to be constant at T_u.)

It was assumed also that the heat flux at the lower boundary of the snowpack equals that from frozen soil to the freezing front. This assumption yields T_0 as:

$$T_0 = \frac{\lambda_s T_a \eta}{\lambda_s \eta + \lambda_f H_s} \tag{8}$$

where T_a is the air temperature; H_s is the depth of snow cover; and λ_f is the thermal conductivity of frozen soil.

The coefficients of thermal conductivity were determined as:

$$\left.\begin{aligned} \lambda_u &= a_\lambda \cdot [2 + \log_{10}(\theta/\rho_u)] + b_\lambda \\ \lambda_f &= \lambda_u \cdot (1 + \theta) \\ \lambda_s &= k_\lambda \cdot \rho_s^2 \end{aligned}\right\} \tag{9}$$

where ρ_u and ρ_s are the densities of soil and snow respectively, and a_λ, b_λ and k_λ are empirical parameters.

The saturated hydraulic conductivity of the frozen soil was calculated as a function of the ice content in the upper 600 mm layer:

$$K_s' = K_s \left[\frac{P - c - \theta_0}{P - \theta_0} \right]^4 / (1 + 8c)^2 \tag{10}$$

where c is the volumetric ice content, determined as the average weighted ice content values in the upper 200 mm layer and in the 200–600 mm layer.

The ice contents in the separate soil layers were determined taking into account the dependence of unfrozen moisture content on the soil temperature.

The infiltration intensity was determined as:

$$I = \begin{cases} K_s', & R' > K_s' \\ R', & R' \leqslant K_s \end{cases} \tag{11}$$

where R' is the intensity of snow melting.

Snow accumulation and snow melting were described in the model, including the processes of the changing of snow density and variations of its water-retention capacity. The degree-day method, used to determine the rate of snow melting, gave:

$$DD = k_m \cdot \rho_s \tag{12}$$

where k_m is an empirical parameter.

Soil thawing was assumed to begin after complete snow melting over the catchment area. The thawing depth was calculated according to an equation similar to equation (6), but without taking into account the impact of the unfrozen soil zone.

To calculate the average flood for the whole basin it was assumed that portions of the basin area with different values of snow water equivalent and soil freezing depth are gamma distributed variables.

The verification of the model was carried out for the Sosna River basin (catchment area 16 300 km^2), located within the Don River basin. The basin is situated in the central part of the forest-steppe zone and is characterized by a large inter-annual variability of snowmelt runoff losses for infiltration, which are caused by differences in the hydrothermal regime of frozen soils. Detailed experimental data are available on the catchment area characteristics and peculiarities of runoff formation.

Numerical experiments on the estimation of the sensitivity of model-computed results to changes of the model parameters showed that the results largely depend on the saturated hydraulic conductivity and the empirical parameter, k_m. Those parameters were calibrated using measured runoff data for the period 1967–1975. The values of those and the other parameters are shown in Table 2. The coefficients of variation of the spatial distribution of snow water equivalent and freezing depth equal 0.45 and 0.90 respectively.

Table 2 *Values of parameters of the model of snowmelt flood formation*

Parameter	*Value*
C_u	0.2 cal g^{-1} °C^{-1}
X	80 cal g^{-1}
ρ_w	1000 g m^{-3}
ρ_u	1420 g m^{-3}
θ_0	0.14
T_u	5°C
a_λ	110 cal m^{-1} s^{-1} °C^{-1}
b_λ	30 cal m^{-1} s^{-1} °C^{-1}
k_λ	0.7 cal m^2 g^{-1} s^{-1} °C^{-1}
k_m	2.0×10^{-8} m^4 g °C^{-1}
K_s	3.0×10^{-6} m s^{-1}
P	0.45

In order to verify the model, measured runoff data for 1965, 1966 and 1977–1981 were used. Figure 8 presents the results of calculations of the spring flood yields for all 16 years. The mean square error of calculations was 13 mm, while the actual average flood yield was 74 mm and the mean square deviation 38 mm.

The input variables of the dynamic model are daily precipitation for the whole year, air humidity deficit for the summer–autumn period and daily air temperature for the winter–spring period. There are many stochastic models of such variables; however their stability and informativeness largely depend on temporal and spatial scales and geographical conditions. As far as is known, only Hanes *et al.* (1977) tried to use a stochastic model of daily precipitation (air temperature and solar radiation were considered practically unchanged from year to year) in developing dynamic-stochastic models of mountain river snowmelt runoff. However, the Hanes model seems too sophisticated; further, its adequacy and stability were not investigated. Developing a model of daily temperature for long periods is most difficult, especially if autocorrelations, varying within wide ranges, are taken into account. Allowing for the difficulties of developing stochastic models of daily air temperature, the following approach was used. First, normalized "fragments" of daily temperature data series for the selected period (season) were obtained from measured data and the observed sequences were divided by their average values. The distribution of the average seasonal temperatures for the whole period of observations was fitted by a gamma distribution with a mean of 4.1°C and C_v = 0.46. Then for the generation of synthetic temperature series,

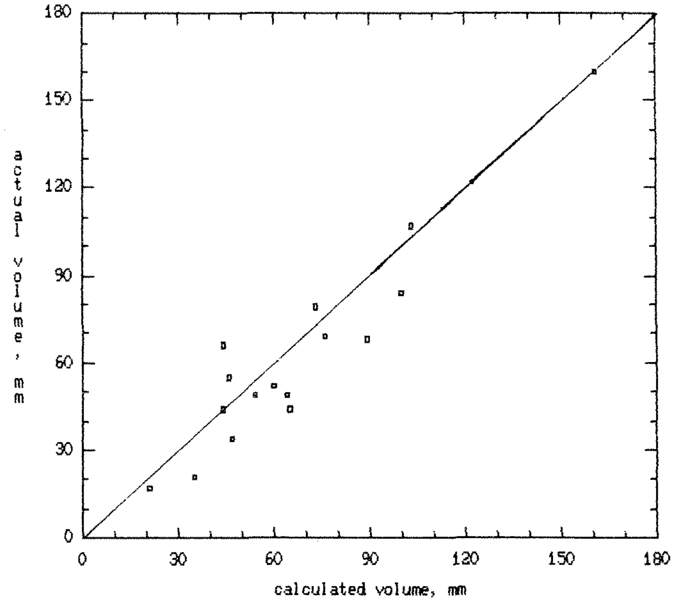

Fig. 8 *Comparison of actual and calculated snowmelt runoff for the Sosna River.*

the random "fragments" were chosen and multiplied by random values of the average temperature determined from the fitted distribution.

The precipitation/non-precipitation sequences were generated for the whole year using a two-state Markov chain. Conditional probabilities were estimated on the basis of meteorological data for the summer–autumn period and, separately, for the winter–spring period. The probability of a dry day after a dry day (P_d) is 0.30 and the probability of a dry day after a wet day (P_w) is 0.60 for the summer–autumn period; for the winter–spring period, $P_d = 0.50$ and $P_w = 0.25$.

For every wet day, random values of the precipitation sum were generated; for every dry day, the average air humidity deficit values (for the summer–autumn period) were also generated. The distributions of those variables were fitted by gamma distributions with parameter values of: mean 4.14 mm, $C_v = 1.31$ for summer precipitation; mean 1.98 mm, $C_v = 1.06$ for winter precipitation; mean of deficit 7.48 mb, $C_v = 0.31$. All those parameters as well as conditional probabilities were estimated using 16 years of data. The histograms of the daily precipitation sum and the air humidity deficit obtained with the observed data and the fitted distributions are shown in Figs 9 and 10. The histograms of the observed durations of periods with precipitation and without precipitation were compared with the histograms of values generated by the Markov chain. The results are shown in Fig. 11.

Using the dynamic model, flood volumes were calculated from the start of May (the initial soil water content was taken to be equal to the field capacity).

The quality of the dynamic-stochastic model of the Sosna River snowmelt flood formation was estimated by comparing the exceedance probabilities of flood volumes calculated using 50 years of observations, and 500 years of generated series (Fig. 12). This Figure shows a satisfactory agreement between

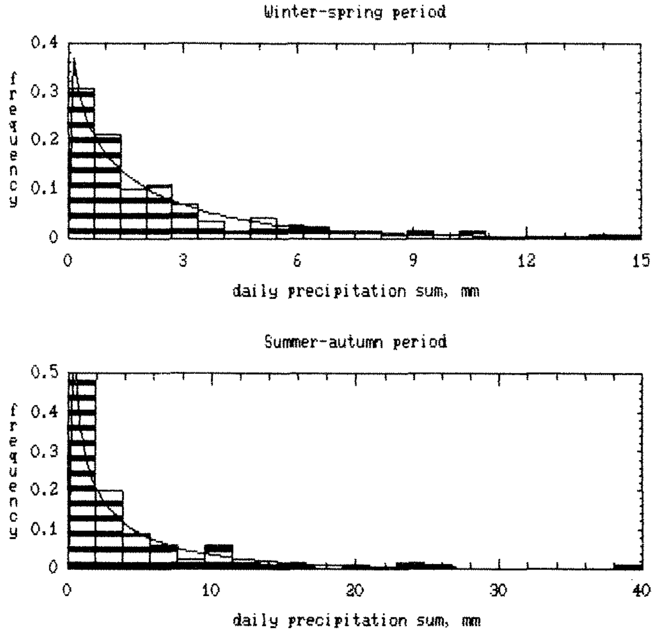

Fig. 9 Histograms of daily precipitation sum values fitted by gamma distributions.

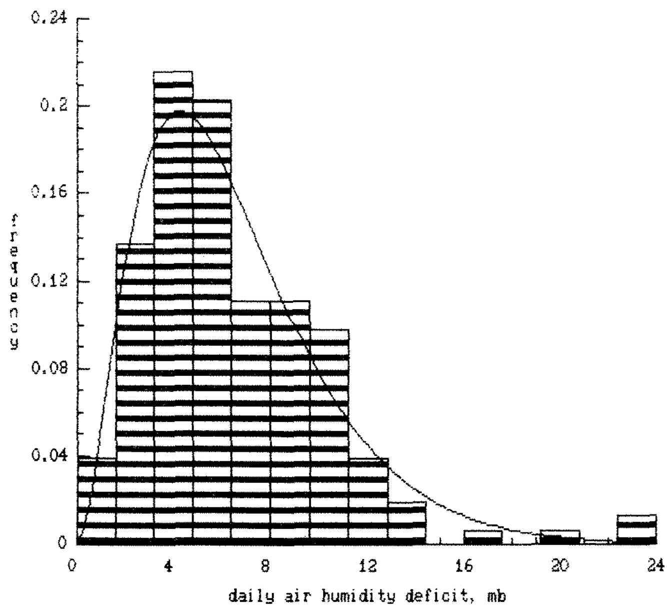

Fig. 10 Histogram of daily air humidity deficit values fitted by a gamma distribution.

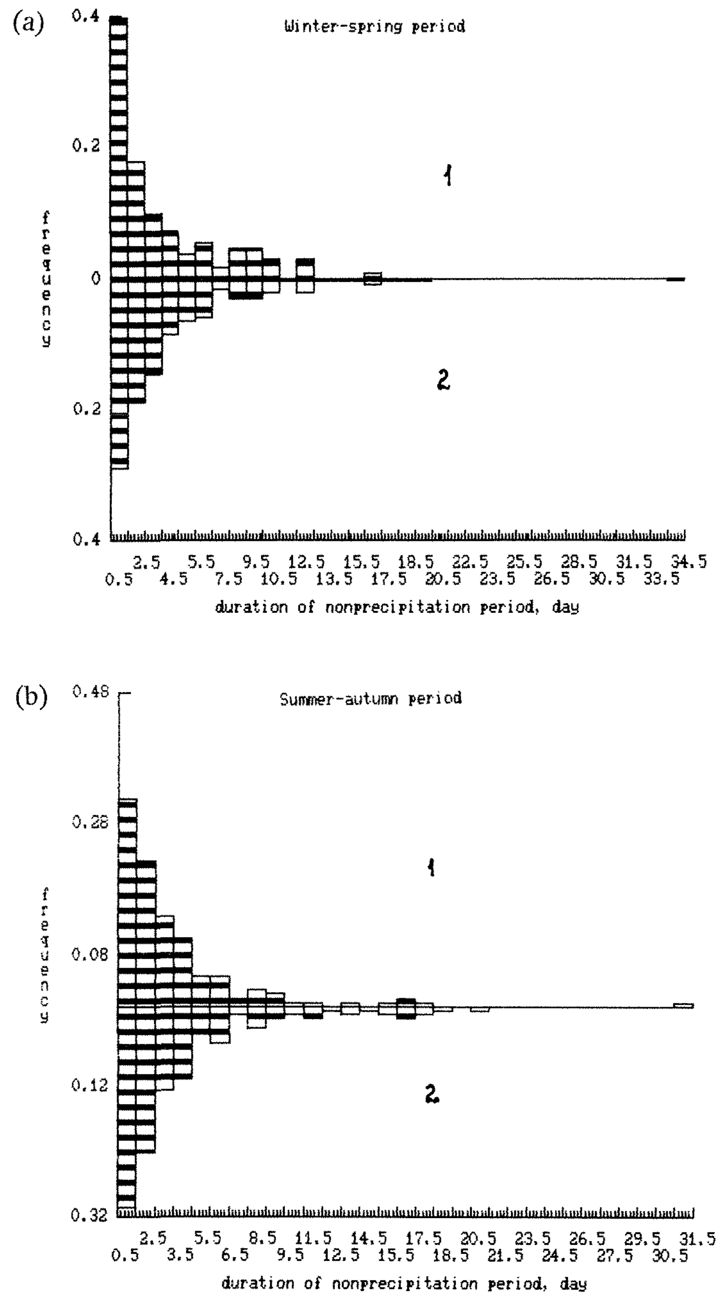

Fig. 11 *Comparison of histograms computed for (1) observed precipitation data and (2) values generated using a Markov chain model: (a) winter–spring period; and (b) summer–autumn period.*

those values despite the fact that the calculations were made applying the stochastic model using a meteorological series three times shorter than the observed runoff data.

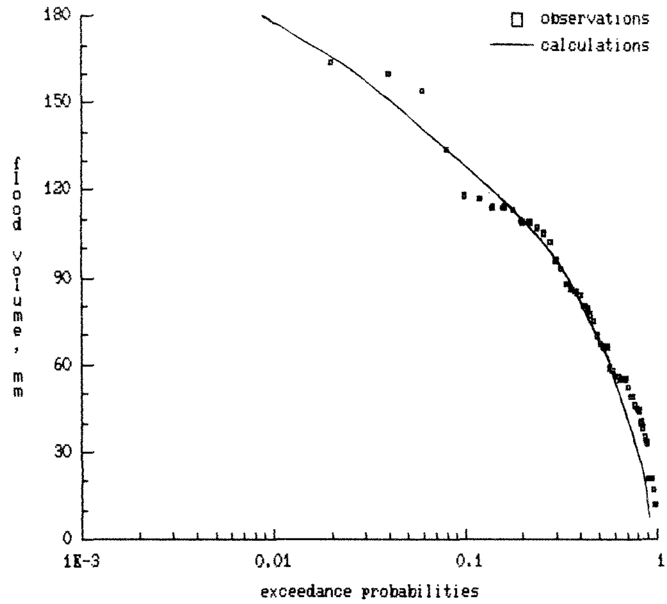

Fig. 12 Comparison of exceedance probabilities of snowmelt runoff calculated according to the model and determined using long-term runoff observations (Sosna River).

CONCLUSION

Application of the Monte-Carlo method combined with numerical methods of solving differential equations describing dynamic processes in hydrological systems leads to the amount of calculations growing very substantially. However, taking into account the increased possibilities of modern computers, the role of those restrictions in the development and use of dynamic-stochastic models is becoming less important. On the other hand, relaxation of restrictions on the quantity of computing operations can lead to unjustified compilations of models from the viewpoint of stability of their parameters and model reliability. The question whether model complication leads to increase in the total amount of information contained in a probabilistic prediction or forecast should be thoroughly analysed, special attention being paid to models of stochastic components. The absence of reliable stochastic models of time series of meteorological values, or insufficient *a priori* information contained in these models, seems to be the main factor restricting the development of dynamic-stochastic models of river runoff formation. However, in a number of cases, even the consideration of possible random combination of observed meteorological impacts on the watershed can ensure a more reliable determination of the statistical distributions of runoff characteristics than the statistical analysis of short observed runoff series. In this connection, accumulation of experience in the development and application of dynamic-stochastic models for different conditions of runoff formation, and various input information, is very important.

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