ISSN 0097-8078, Water Resources, 2018, Vol. 45, Suppl. 1, pp. S11–S17. © Pleiades Publishing, Ltd., 2018.

Numerical Modeling of Non-Uniform Sediment Transport in River Channels

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Received July 2, 2018

Abstract—The mathematical model for simulating deformations of river channels composed of heterogeneous alluvium has been developed. The combination of shallow water equations and a three-layer model is used to describe the fluid flow and non-uniform sediment transport in bed (layer II) and suspended (layer III) loads. Changes in the fractional composition of unerodible bottom sediments (layer I) are also considered. The algorithm provides mass conservation for each fraction. The comparison of calculations results and experimental data (hydraulic washing of a desilting basin from sediments and armoring processes in heterogeneous soils) confirms the operability of the model. The model is applied to calculate the silting and hydraulic washes of the reservoir of a hydroelectric power station on a mountain river.

Keywords: numerical modeling, channel deformations, heterogeneous sediments, armoring

DOI: 10.1134/S0097807818050275

INTRODUCTION

Sediment grainsize may differ by several orders of magnitude in rivers of mountain and upland regions, so it is necessary to take this fact into account when modeling channel deformations. In this case simulations based only on a mean diameter can lead to qualitatively and quantitatively incorrect results, which do not correspond to the physics of the phenomenon under consideration. The heterogeneity of the soil is especially pronounced when calculating the silting of reservoirs, which occurs both as a result of the flow of bed-forming fractions and the deposition of suspended loads of fine fractions (in the domestic conditions they are transferred along the river). Large errors are made not only in the forecast of intensity and structure of silting, but especially in calculating the efficiency of hydraulic washes of reservoirs that are carried out to maintain the total storage and effective volume of the reservoir in a given range. Loads with different sizes have different mobility and are therefore deposited and washed away in different parts of the reservoir, depending on the depths and flow velocities. In this case, the armouring effect can be manifested, when in the process of bottom erosion large fractions do not allow smaller fractions to enter the water flow, thereby slowing down the intensity of erosion and worsening the results of washing the reservoir.

The three-layer mathematical model proposed in this article is based on a previously developed and well-proven model for closely graded sediments [4, 5]. The main physical processes of sediment transport by a turbulent flow are taken into account in the approximation of shallow water: the transport of different sediment fractions in bed load and suspended forms (in the main stream and near the bottom); the suspension and sedimentation of soil particles; the transport of sediments in the direction transverse to the flow velocity vector (diffusion of underwater and overwater slopes); the influence of the Froude number on the intensity of the bottom deformations; changes in the fractional composition of bottom sediments during the calculation; armoring of heterogeneous deposit. The model provides mass conservation for each fraction and makes it possible to calculate not only bottom deformations, but also the erosion of bank slopes, as well as calculations of the dam-break process during hydrodynamic failures. The approach is in line with current trends in the numerical simulation of flows with a deformable bottom in a plane problem formulation (see, for example, [6, 7, 10–12]). The numerical algorithm for the solution of shallow water equations is based on the developed solver of the Riemann problem over a discontinuous bottom [1, 3].

To demonstrate the efficiency of the developed model three laboratory experiments [8, 9] and a real practical problem are considered (accumulation and hydraulic washes of the Krasnogorsk reservoir located on a mountain river). For all the cases numerical

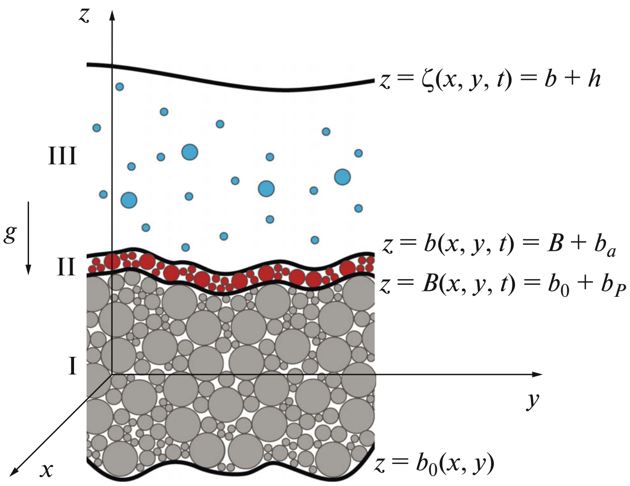

Fig. 1. Schematic representation of three layers of non-uniform soil.

results show that the model works well in a wide range of parameters.

MATHEMATICAL MODEL

The fluid motion in the shallow water approximation is described by the two-dimensional Saint-Venant equations (in the horizontal plane) in the Cartesian coordinate system (x,y)

$$h_{,t} + \nabla \cdot (h\boldsymbol{u}) = 0, \quad (1)$$

$$(h\boldsymbol{u})_{,t} + \nabla \cdot (h\boldsymbol{u}\boldsymbol{u}) + gh\nabla(h+b) = \boldsymbol{F}. \quad (2)$$

Here t is time; $h = h(x,y,t)$ is the depth of a fluid flow; $\boldsymbol{u} = \boldsymbol{u}(x,y,t) = (u,v)^T$ is the depth-averaged velocity vector; g is the acceleration due to gravity (Fig. 1); $b = b(x,y,t)$ is surface of bottom; $\zeta(x,y,t) = h + b$ is free surface; $\boldsymbol{F}$ describes the action of external forces, for example, the friction force: $\boldsymbol{F} = \lambda\boldsymbol{u}|\boldsymbol{u}|/2$, where $\lambda = 2gn^2h^{-\frac{1}{3}}$ and n are coefficients of hydraulic friction and roughness; $(\cdot)_{,t} = \partial/\partial t$; $\nabla = (\partial/\partial \mathrm{x}, \partial/\partial \mathrm{y})$.

To describe the motion of a soil consisting of N fractions, the following model is proposed. It is assumed that the load carrying capacity of the flow and the flow rate at which the particles of the fraction begin to move can be determined independently for each fraction. In addition, it is considered that the soil is uniformly mixed in accordance with the calculated fractions concentrations. From the first assumption in particular it follows that the fractions cannot have close characteristics, because in this case the carrying capacity of the flow cannot be determined independently. According to experimental data [9], the soil in the calculations can approximately be considered homogeneous with the ratio of the diameter of the largest fraction to the smallest diameter less than 5.

Three layers are considered (Fig. 1):

I. **Passive layer** is a fixed bottom layer of soil that lies on the surface of the unerodible bed $z = b_0(x,y)$. Its fractional composition is determined by the contributions of b_p^f of each fraction to the total depth of this layer $b_p = \sum_{f=1}^{N} b_p^f$, $f = 1,2,\ldots,N$ is a fraction number.

II. **Active soil layer** is located above the passive layer $z = B(x,y,t) = b_0 + b_p$. The movement of the soil in the active layer occurs due to the movement of the bed loads and the process of diffusion of the bottom. The thickness of the layer is determined by the formula

$$b_a = \begin{cases} \max\left(k_D D_{50}^{\max}, k_U\left(|\boldsymbol{u}| - U_N^{\max}\right)^2\right), & \text{if} |\boldsymbol{u}| \ge U_N^{\max} \\ k_D D_{50}^{\max}, & \text{if} |\boldsymbol{u}| < U_N^{\max} \end{cases}, \quad (3)$$

where k_D, k_U are given constants; $D_{50}^{\max}$ is particle diameter of 50% probability for the largest fraction; $U_N^{\max}$ is non-moving speed of the largest fraction.

In the case when the largest fraction is stationary ($|\boldsymbol{u}| < U_N^{\max}$), the depth of the moving layer is determined by the average diameter for the largest fraction, thus the effect of armoring is simulated. Therefore, the small fraction can suspend until its concentration in the active layer becomes zero. At this point, in spite of the presence of a fine fraction on the bottom in the passive layer, the thin layer of larger particles prevents the process of its suspension. At sufficiently high velocities ($|\boldsymbol{u}| > U_N^{\max}$), it is necessary to take into account the dependence of the depth of the moving layer on the tangential stresses occurring on the surface of the active layer. In the present paper, this is done using the expression $k_U\left(|\boldsymbol{u}| - U_N^{\max}\right)^2$ in formula (3), which is also a proportionality coefficient in the empirical formula for determining the equilibrium concentration (see formula (8)).

The contribution to the thickness b_a of each fraction is denoted by b_a^f, i.e., $b_a = \sum_{f=1}^{N} b_a^f$.

III. **Suspended (and saltation) loads** are particles that are transported in a fluid flow bounded by surfaces $z = b(x,y,t)$ and $z = \zeta(x,y,t)$. The concentration of each fraction in the flow is denoted by S^f.

Let us consider the relations describing the processes of mass exchange between layers I, II (through the surface $z = B(x,y,t)$) and II, III (through the surface $z = b(x,y,t)$). The analysis of degenerate cases, when the layers have zero depth, is omitted, for greater clarity of the approach presented.

The mass flow at boundary I–II occurs when a part of the passive layer passes into the active layer or vice versa. These transitions are described by the relations expressing the law of conservation of mass for each fraction.

$$\left(b_p^f\right)_{,t} = \frac{1}{2}\left(B_{,t} - \left|B_{,t}\right|\right)\frac{b_p^f}{b_p} + \frac{1}{2}\left(B_{,t} + \left|B_{,t}\right|\right)\frac{b_a^f}{b_a}, \tag{4}$$

$$b^f = b_p^f + b_a^f, \tag{5}$$

$$B = b - b_a,\ b = b_0 + \sum_{f=1}^{N} b^f. \tag{6}$$

The mass flow at the boundary II–III is associated with the processes of suspension and sedimentation of particles. It is assumed that for the fraction with number f the particle mass flux at the boundary $z = b(x,y,t)$ is described by the expression

$$F_w^f = K^f\left(S^f - S_e^f\right), \tag{7}$$

where S_e^f is the equilibrium concentration under saturation conditions; K^f is the coefficient of vertical sediment exchange between the bottom and the flow (at $z = b(x,y,t)$), which depends on the settling velocity of the sediments fraction and the dynamic flow velocity. If $F_w^f > 0$ ($F_w^f < 0$), the particles sediment (suspend). For the equilibrium concentration, S_e^f the following formula is used

$$S_e^f = \begin{cases} \alpha \dfrac{\rho_w \lambda \left(\left|\boldsymbol{u}\right| - U_N^f\right)^2}{2\rho_s^f g h}\left(\dfrac{0.13}{\tan\varphi^f} + \dfrac{0.01\left|\boldsymbol{u}\right|}{W^f}\right), & \text{if} \left|\boldsymbol{u}\right| > U_N^f; \\ 0, & \text{if} \left|\boldsymbol{u}\right| \le U_N^f \end{cases} \tag{8}$$

where ρ_s^f, ρ_w are the density of fraction f and the fluid; α is the empirical coefficient, which depends on the Froude number; W^f is settling velocity; $\tan\varphi^f$ is tangent of the angle of the depositional gradient of the soil in water. The non-moving velocity is determined from relation (a modified Goncharov formula, in which, under the sign of the logarithm in the numerator, a particle diameter of 90% probability is added for the possibility of computations by the formula for h tending to zero)

$$U_N^f = \log\left[8.8\left(\frac{h + D_{90}^f}{D_{90}^f}\right)\right]\sqrt{\frac{4}{7}\left(\frac{\rho_s^f}{\rho_w} - 1\right) g D_{50}^f}, \tag{9}$$

where D_{50}^f, D_{90}^f are particle size characteristics: particle diameters of 50 and 90% probability for the corresponding fraction.

Thus, the shallow water equations (1), (2) are supplemented by the equations describing the transport of particles in the flow, their suspension and sedimentation, and diffusion of the bottom.

$$\left(hS^f\right)_{,t} + \nabla \cdot \left(\boldsymbol{u} h S^f\right) = -F_w^f, \tag{10}$$

$$(1 - p)\, b_{,t}^f = F_w^f + \nabla \cdot \left(D^f \nabla b\right), \tag{11}$$

where the superscript f determines the fraction number and changes from 1 to N; D^f is the diffusion coefficient for the fraction with number f; p is soil porosity. The process of diffusion of the bottom is described by the expression $\nabla \cdot \left(D^f \nabla b\right)$, which takes into account diffusion due to bedload flow, 'slipping' soil in water and on land (the tendency to form an angle of the slope that is not greater than natural one).

In the described model, there are $5 + 4N$ unknown scalar functions ($u, v, h, B, b, b_a^f, b_p^f, b^f, S^f$), with the same amount of equations: $3 + 3N$ differential equations (1), (2), (4), (10), (11) and $2 + N$ algebraic relations (5), (6). Initial-boundary value problems for the described system of equations are solved by a numerical method of the Godunov type on unstructured triangular-quadrangular meshes, with mesh refinement in regions with complex boundary geometry. To solve the Saint-Venant equations in the presence of a complex bottom topography (including discontinuities), a specially developed algorithm for solving the Riemann problem with bottom discontinuity is used [1, 3]. The presented algorithm is implemented for parallel computations on NVIDIA graphics processors (using CUDA – Compute Unified Device Architecture) and it is integrated into the STREAM 2D software package [2].

RESULTS

To verify the model, a comparison of the calculations with the data of laboratory experiments with homogeneous and heterogeneous sediments was carried out. A verification of Goncharov's formula for critical velocity for homogeneous deposit was carried out in independent data (V.S. Knoroz's [9] and Z.D. Kopaliani's [8] experiments for a homogeneous non-cohesive soil). The data array was represented in a wide range of variation in the relative depth and the dimensionless flow rate. Figure 2 shows that values obtained by the Eq. (9) completely coincide with logarithmic approximation of the results of the experiments.

The calculations were also compared with experiments simulating the washing of a desilting basin from loads deposited therein. The experiments were carried out in Joint Stock Company 'Research Institute of Power Structures' (JST 'RIPS') in 2007 in a flume with a length of 16 m and a width of 0.54 m. Initially,

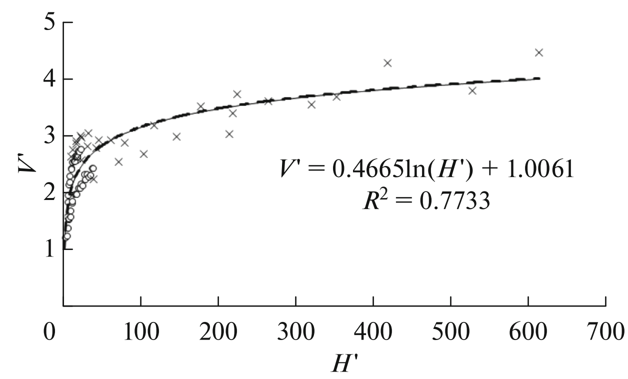

Fig. 2. Logarithmic approximation of the results of the experiments [8, 9] for homogeneous soils. Here $H' = (h + D_{90})/D_{90}$, $V' = V_N \Big/ \sqrt{\left(\frac{\rho_s}{\rho_w} - 1\right) g D_{50}}$ are dimensionless depth and velocity. Critical and non-moving (Eq. (9)) velocities are related by $V_N = \sqrt{2} U_N$. The circles and crosses are data obtained from experiments [8] and [9]; the solid line corresponds to the critical velocity obtained using Eq. (9); the dashed line represents the logarithmic trend.

the desilting basin was silted in 70% of the length by a uniform fine sand of medium size 0.15 mm. The settler was washed with a water flow of 0.031 m^3/s for 45 min, during which all the sediment from it was removed. The dynamics of erosion is very interesting. Initially, the only steep bottom slope of the deposits gradually split into several sand waves, which in a certain time interval (from about 15 to 20 min) began to move upward against the current, i.e. in the so-called ‘antidune’ regime. Exactly the same effect was observed in a numerical experiment. Longitudinal profiles of the surface of the deformable bottom in the calculation and experiment for some instants of time are shown in Fig. 3.

To verify the correctness of the description of the armoring during the erosion of a mixture of soils of different sizes, the results of Knoroz [9] were reproduced on a numerical model. The laboratory experiments [9] were performed in a flume with a length of 9.8 m and a width of 0.63 m for a wide range of soil parameters, Froude numbers, velocities and flow depths. The ratio of the diameters of large and fine fractions varied from 6.0 to 27.4, the percentage of large fraction in the soil composition varies from 10 to 40%, the Froude number varies from 0.08 to 0.93. Experiments (both physical and numerical) were carried out until the erosion depth was completely stabilized. A comparison of the calculated and measured depths of the flow after erosion is shown in Fig. 4 and demonstrates a sufficiently high accuracy of the calculations.

Modeling of silting and hydraulic washes of the designed Krasnogorsk reservoir in the upper reaches of the Kuban River in the mountainous terrain has been performed for a period of up to 50 years. The calculations took into account the real long-term hydrograph of the Kuban River in the studied cross-section for the past years. At the input boundary of the model the connection between the sediment concentration in the flow and the water discharge is used, which is sufficiently well substantiated by actual data. A soil with a particle size from 0.02 to 200 mm was considered (the range of variation of the diameter was 4 orders of magnitude), which was divided into 6 fractions with average diameters of 0.05, 0.25, 1 mm (suspended load) and 5, 20, 100 mm (bed load).

The simulations of the silting process and, in particular, the washing of the reservoir show qualitative differences between models with homogeneous and essentially heterogeneous sediments. For a homogeneous soil, periodic washing of the reservoir results in quasi-stationary operating regime, when the total storage and effective volumes practically ceased to change during a year. For heterogeneous soil under the same operating conditions, the full and useful volumes continued to decrease throughout the calculation period, which is a negative factor and requires additional measures to stabilize the silting process (catching pit for additional deposition of sediments with their subsequent removal, dredging, etc.).

Figure 5 shows the dynamics of siltation of the reservoir after 7 and 17 years of operation with annual hydraulic washes, and zones of sedimentation of small, medium and large fractions in 17 years of the reservoir’s existence. It can be seen that small fractions (which are transported in a suspended state in domestic conditions) are deposited near the dam of the hydrosystem, where depth is large and flow velocity is small. Large fractions are deposited mainly in the tail part of the reservoir. Performing hydraulic flushing of the reservoir from the first year of operation ensures the transit of sediment to the tail water through a deep spillway and practically prevents them from entering the waterworks of the hydro power station, which helps to reduce the wear of the turbine blades.

Table 1 shows a comparison of the fractional composition of the sediments entering the river and passing through the spillway after 17 years of the operation of the hydroelectric unit with annual washes. It can be seen that the percentage of fine fractions transferred through the site of the hydrounit is significantly increased in comparison with the initial concentration in the flow, and the percentage of the large fractions is significantly reduced, i.e. they are mainly deposited in the reservoir. Figure 6 shows the change in time of the mass of sediment by fractions passing through the operational spillway (the silting of the reservoir with annual washes). The graphs have a stepped view, because the transport of sediment through the spillway occurs mainly in short-term periods of hydraulic washes. All fractions pass through the spillway, includ-

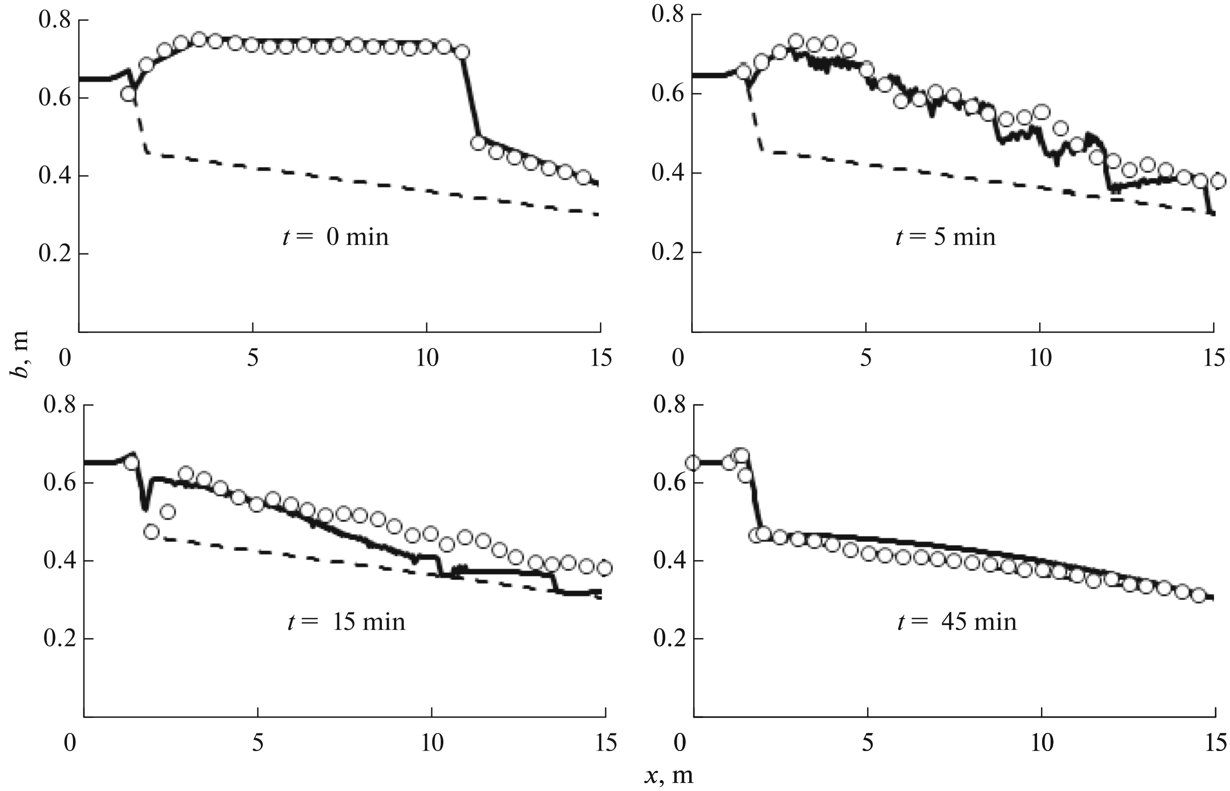

Fig. 3. Longitudinal profiles of the surface of the deformable bottom in the calculation and experiment at t = 0, 5, 15, 45 min. The circles are experimental data; the solid lines correspond to calculation; the dashed lines represent nonerosive bottom.

ing the largest one with a size D_{50} = 100 mm (fraction 6). It should be noted that during the operation of the reservoir without flushing through the hydro power station, almost 100 times more deposits of two small fractions are carried, and after 14 years of operation, deposits of a size of 1 mm or more begin to flow into the turbine tract, which will lead to accelerated wear of the turbines.

CONCLUSIONS

The proposed model describing the non-uniform sediment transport is based on the representation of the soil in the form of three interacting layers: the stationary layer (allowing the flow of mass through its surface); the active layer from the bed loads, and the layer carried by the main body of the flow. The fluid flow is described by the shallow water equations, which allows to effectively model the extended sections of a river and a reservoir, which is important for solving practical problems. The developed model is applied to the three laboratory experiments and a real practical problem. It is shown that the critical velocity and the dynamics of the washing process for one fraction are consistent with the laboratory experimental data. Based on the experimental data of Knoroz [9], it is shown that, in the presence of several fractions in the flow, the developed model works well in a wide range of parameters. Plausible results of modeling a practical problem with the accumulation and hydraulic washes

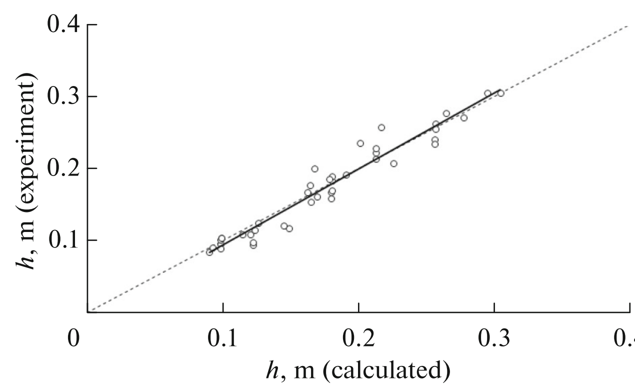

Fig. 4. Comparison of calculated and measured [9] flow depths, which indicate the maximum erosion level of a non-uniform bed. The circles show the depth of erosion; the solid line is the linear trend; the dashed line shows the bisector.

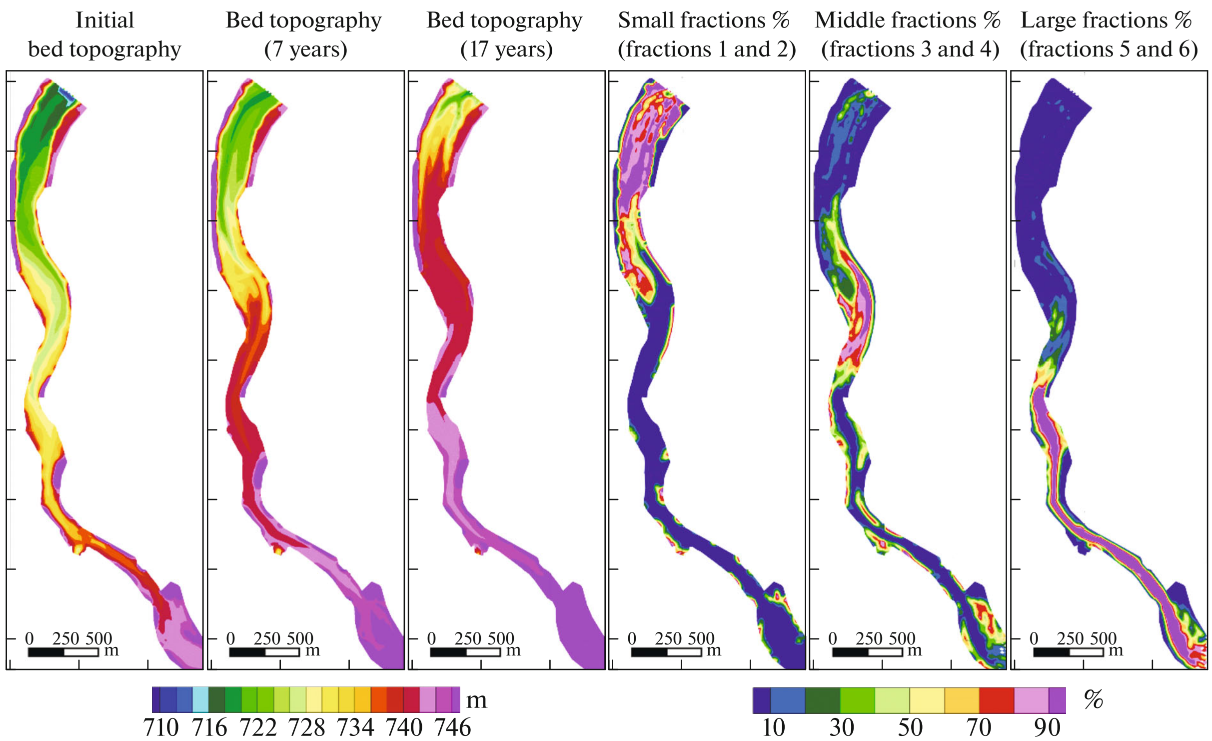

Fig. 5. Bed topography (3 plots to the left) during the silting of the Krasnogorsk reservoir with annual washes and fractional composition (3 plots to the right) of bed sediments at the end of calculation (in 17 years).

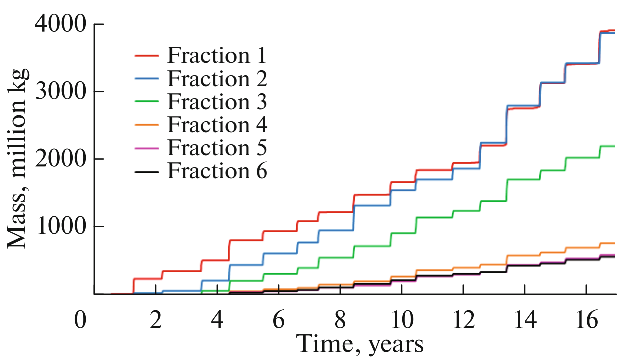

Fig. 6. Fractions mass of the sediments carried out through the spillway in calculation with annual washes.

Table 1. Fractional composition of the soil in modeling the siltation of the reservoir with annual washes at the end of the calculation period

Fraction	D_{50}, mm	D_{90}, mm	Soil composition, %	
			inflow boundary	spillway
Fraction 1	0.05	0.15	24	32.85
Fraction 2	0.25	0.75	22	32.51
Fraction 3	1.00	3.00	20	18.49
Fraction 4	5.00	15.00	8	6.43
Fraction 5	20.00	60.00	13	4.98
Fraction 6	100.00	300.00	13	4.75

of a reservoir located on a mountain river are obtained as well.

ACKNOWLEDGEMENTS

The authors are very grateful to the former employee of the Center for Hydraulic Research of the JSC 'RIPS' V.P. Seredavkin for the materials of experimental research, as well as to the staff of the Center who participated in the experiments. The work was supported by the Russian Science Foundation, project no. 17-77-30006.

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